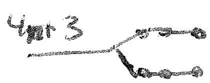
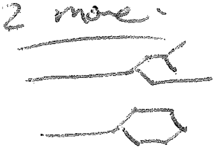
Classification program: (small index)

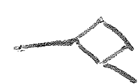
(1) Enumerate
(2) Obstruct
(3) Construct

] Sometimes do this part together.

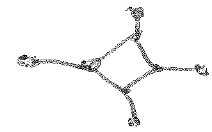
Index	(1)	(2)	(3)
< 4	$ADE_{6,7,8}$ (Kronecker)	No D_{odd}, E_7	Jones, Kawahigashi, Bion-Nadal
= 4 = $3+\sqrt{1}$	Affine Dynkin	all OK	GHJ, Izumi-K, Popa
$3+\sqrt{2}$	4 or 3  Line	triple pt (Ocneanu's), AY	AH, BMPS
$3+\sqrt{3}$	2 more: 	- advanced triple pt (Haagerup / MPPS) - Bisch	—
< 5 = $3+\sqrt{4}$	 + weeds	- weeds: pt 2 MPPS, pt 3 IJMS, pt 4 PT (CMS) ← generalizes AY; - vines: pt 1 MS	GHJ, IX

at 5: known, we have to write the paper.

Beyond?

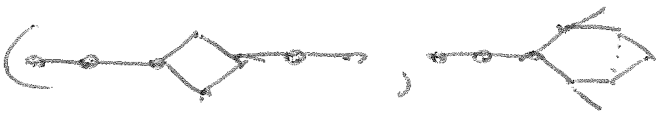
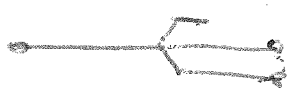
MPt: - only 1 1ST w/ index in $(5, 3+\sqrt{5})$ 
- any other SF w/ index in $(5, 3+\sqrt{5})$ has rank > 39.

~~BMPP~~ Liu: 1ST at index $3+\sqrt{5}$. (1) BH enumerated possible 7 1ST std. invariants; (1)+(3): Liu: c.d. GPA.

LMP: 1ST below $6\frac{1}{5}$: either has intermediate, or ∄ in $(3+\sqrt{5}, 6\frac{1}{5})$, etc, is  $(3^{\mathbb{Z}})$.

⇝ Liu's $E_{\pm 12} \subset SU(N)$ quantum subgp
(these indices grow as $N \to \infty$)

At index $3+\sqrt{5}$:

1. 7 1-ST SFPA's (Liu / IMP is 3rd fish)
2. () unique (P)
   (only 1 SF w/  principal graph)
   – unify paper w/ Scott about 2D2 + Quantum Subgps.
3. $3^{\mathbb{Z}_2 \times \mathbb{Z}_2}$, $3^{\mathbb{Z}/4}$ unique. (Izumi, MP, PP)
4. 4442  equi of $3^{\mathbb{Z}_2 \times \mathbb{Z}_2} \circlearrowleft \mathbb{Z}/3$

   Should this $\mathbb{Z}/3$ lift to $\mathbb{Z}$?

Q: How many 4442's are there?
Conj: only 1.

Conj: That's it at $3+\sqrt{5}$.

Thm: Infinitely many SF at $PA(G, 5^2)$?

Index 5 to $5\frac{1}{4}$: work w/ Afzaly + Morrison.

1. weeds are $\chi 10$, $\chi 11$ ⟵ new 3-pt obstruction (P) does this
   one w/ $r=1$ (that's bad), others ✓
   – stable weeds are cylinders – finitely many of these to rule out
   – others.

Thm (Wed/Thurs) $\chi 1$ weed is dead. ‖ Thm: (Calegari's Sturbino) – cylinders finite problem