(1a) For every closed (compact, smooth, oriented) 1-manifold S, a category $\mathcal{C}(S)$, isomorphic to $\mathsf{Vec}_{\text{f.d.}}^{\oplus r}$ for some $r \in \mathbb{N}$ which depends on S.

[Think: There is a certain group or algebra associated to S, and $\mathcal{C}(S)$ is the category of representations of that group or algebra (r = number of irreps.)]

For every pair of 1-manifolds S_1, S_2 there is a bilinear functor $\mathcal{C}(S_1) \times \mathcal{C}(S_2) \to \mathcal{C}(S_1 \sqcup S_2) : (\lambda, \mu) \mapsto \lambda \otimes \mu$ which induces an equivalence of categories

$$\mathcal{C}(S_1) \otimes \mathcal{C}(S_2) \xrightarrow{\cong} \mathcal{C}(S_1 \sqcup S_2).$$

Here, given two linear categories $\mathcal{C}$ and $\mathcal{D}$ isomorphic to $\mathsf{Vec}_{\text{f.d.}}^{\oplus r}$, their tensor product $\mathcal{C} \otimes \mathcal{D}$ has objects of the form $\bigoplus c_i \otimes d_i$ for $c_i \in \mathcal{C}$ and $d_i \in \mathcal{D}$, and hom-spaces given by $\mathrm{Hom}_{\mathcal{C}\otimes\mathcal{D}}(\bigoplus c_i \otimes d_i, \bigoplus c'_j \otimes d'_j) = \bigoplus_{ij} \mathrm{Hom}_{\mathcal{C}}(c_i, c'_j) \otimes \mathrm{Hom}_{\mathcal{D}}(d_i, d'_j)$.

We also have an equivalence $\mathsf{Vec}_{\text{f.d.}} \xrightarrow{\cong} \mathcal{C}(\emptyset) : \mathbb{C} \mapsto 1$.

There is an associator $(\lambda \otimes \mu) \otimes \nu \xrightarrow{\cong} \lambda \otimes (\mu \otimes \nu)$, unitors $1 \otimes \lambda \xrightarrow{\cong} \lambda$ and $\lambda \otimes 1 \xrightarrow{\cong} \lambda$, and a braiding $\lambda \otimes \mu \xrightarrow{\cong} \mu \otimes \lambda$ *[we omit the isomorphisms $(S_1 \sqcup S_2) \sqcup S_3 \cong S_1 \sqcup (S_2 \sqcup S_3)$, $\emptyset \sqcup S \cong S$, $S \sqcup \emptyset \cong S$, and $S_1 \sqcup S_2 \cong S_2 \sqcup S_1$]* which are natural (i.e. for any morphisms $\lambda \to \lambda'$, $\mu \to \mu'$, $\nu \to \nu'$ the following diagrams commute

$$\begin{tikzcd}
(\lambda\otimes\mu)\otimes\nu \arrow[r] \arrow[d] & (\lambda'\otimes\mu')\otimes\nu' \arrow[d] \\
\lambda\otimes(\mu\otimes\nu) \arrow[r] & \lambda'\otimes(\mu'\otimes\nu')
\end{tikzcd}
\quad
\begin{tikzcd}
1\otimes\lambda \arrow[r] \arrow[d] & 1\otimes\lambda' \arrow[d] \\
\lambda \arrow[r] & \lambda'
\end{tikzcd}
\quad
\begin{tikzcd}
\lambda\otimes 1 \arrow[r] \arrow[d] & \lambda'\otimes 1 \arrow[d] \\
\lambda \arrow[r] & \lambda'
\end{tikzcd}
\quad
\begin{tikzcd}
\lambda\otimes\mu \arrow[r] \arrow[d] & \lambda'\otimes\mu' \arrow[d] \\
\mu\otimes\lambda \arrow[r] & \mu'\otimes\lambda'
\end{tikzcd}
\quad)$$

and subject to the well-known pentagon, triangle, hexagon, and symmetry axioms (the same axioms which appear in the definition of a symmetric monoidal category):

$$\begin{tikzcd}[column sep=small]
& (\lambda\otimes\mu)\otimes(\nu\otimes\rho) \arrow[dr] & \\
((\lambda\otimes\mu)\otimes\nu)\otimes\rho \arrow[ur] \arrow[d] & & \lambda\otimes(\mu\otimes(\nu\otimes\rho)) \\
(\lambda\otimes(\mu\otimes\nu))\otimes\rho \arrow[r] & \lambda\otimes((\mu\otimes\nu)\otimes\rho) \arrow[ur] &
\end{tikzcd}
\quad
\begin{tikzcd}[column sep=small]
& \lambda\otimes\mu \arrow[dr] & \\
(\lambda\otimes 1)\otimes\mu \arrow[ur] \arrow[rr] & & \lambda\otimes(1\otimes\mu)
\end{tikzcd}
\quad
\begin{tikzcd}[column sep=small]
& (\mu\otimes\lambda)\otimes\nu \arrow[r] & \mu\otimes(\lambda\otimes\nu) \arrow[dr] & \\
(\lambda\otimes\mu)\otimes\nu \arrow[ur] \arrow[dr] & & & \mu\otimes(\nu\otimes\lambda) \\
& \lambda\otimes(\mu\otimes\nu) \arrow[r] & (\mu\otimes\nu)\otimes\lambda \arrow[ur] &
\end{tikzcd}
\quad
\begin{tikzcd}[column sep=small]
& \mu\otimes\lambda \arrow[dr] & \\
\lambda\otimes\mu \arrow[ur] \arrow[rr, "="] & & \lambda\otimes\mu
\end{tikzcd}$$

(1b) For every closed 1-manifold S, a faithful functor $U : \mathcal{C}(S) \to \mathsf{TopVec}$ which equips $\mathcal{C}(S)$ with the structure of a concrete category.

If $\mathcal{C}(S) \cong \mathsf{Vec}_{\text{f.d.}}^{\oplus r}$, so that an object can be written as an r-tuple of finite dimensional vector spaces, then the U functor is always of the form $(V_1, \ldots, V_r) \mapsto \bigoplus V_i \otimes W_i$, where the W_i are typically infinite dimensional.

The forgetful functor satisfies $U(\lambda \otimes \mu) = U(\lambda) \otimes U(\mu)$ and $U(1) = \mathbb{C}$, naturally in λ and μ, and compatibly with the associator, unitors, and braiding:

$$\begin{tikzcd}
U((\lambda\otimes\mu)\otimes\nu) \arrow[r, phantom, "="] \arrow[d] & (U(\lambda)\otimes U(\mu))\otimes U(\nu) \arrow[d] \\
U(\lambda\otimes(\mu\otimes\nu)) \arrow[r, phantom, "="] & U(\lambda)\otimes(U(\mu)\otimes U(\nu))
\end{tikzcd}
\quad
\begin{tikzcd}
U(1)\otimes U(\lambda) \arrow[r, phantom, "="] \arrow[d, equal] & U(1\otimes\lambda) \arrow[d] \\
\mathbb{C}\otimes U(\lambda) \arrow[r] & U(\lambda)
\end{tikzcd}
\quad
\begin{tikzcd}
U(\lambda)\otimes U(1) \arrow[r, phantom, "="] \arrow[d, equal] & U(\lambda\otimes 1) \arrow[d] \\
U(\lambda)\otimes\mathbb{C} \arrow[r] & U(\lambda)
\end{tikzcd}
\quad
\begin{tikzcd}
U(\lambda\otimes\mu) \arrow[r, phantom, "="] \arrow[d] & U(\lambda)\otimes U(\mu) \arrow[d] \\
U(\mu\otimes\lambda) \arrow[r, phantom, "="] & U(\mu)\otimes U(\lambda)
\end{tikzcd}$$

(2a) For every complex cobordism with thin parts Σ from S_1 to S_2, a linear functor $F_\Sigma : \mathcal{C}(S_1) \to \mathcal{C}(S_2)$.

If Σ and Σ' are complex cobordisms from S_1 to S_2, then for every biholomorphic map $\phi : \Sigma \xrightarrow{\cong} \Sigma'$ such that $\phi|_{S_1} = \mathrm{id}$ and $\phi|_{S_2} = \mathrm{id}$, we have an invertible natural transformation $F_\Sigma \cong F_{\Sigma'}$, compatible with composition of maps.

We also have invertible natural transformations $F_{\mathbf{1}_S} \cong \mathrm{id}_{\mathcal{C}(S)}$, $F_{\Sigma_1 \cup \Sigma_2} \cong F_{\Sigma_1} \circ F_{\Sigma_2}$, and $F_{\Sigma_1 \sqcup \Sigma_2} \cong F_{\Sigma_1} \otimes F_{\Sigma_2}$. They are natural with respect to biholomorphic maps of complex cobordisms, and make the following diagrams commute:

$$\begin{tikzcd} F_{\mathbf{1}_S \cup \Sigma} \arrow[r] \arrow[d] & F_{\mathbf{1}_S} \circ F_\Sigma \arrow[d] \\ F_\Sigma \arrow[r, equal] & \mathrm{id}_{\mathcal{C}(S)} \circ F_\Sigma \end{tikzcd} \quad \begin{tikzcd} F_{\Sigma \cup \mathbf{1}_S} \arrow[r] \arrow[d] & F_\Sigma \circ F_{\mathbf{1}_S} \arrow[d] \\ F_\Sigma \arrow[r, equal] & F_\Sigma \circ \mathrm{id}_{\mathcal{C}(S)} \end{tikzcd} \quad \begin{tikzcd} F_{\mathbf{1}_{S_1} \sqcup \mathbf{1}_{S_2}} \arrow[r] \arrow[d] & F_{\mathbf{1}_{S_1}} \otimes F_{\mathbf{1}_{S_2}} \arrow[d] \\ \mathrm{id}_{\mathcal{C}(S_1 \sqcup S_2)} \arrow[r] & \mathrm{id}_{\mathcal{C}(S_1)} \otimes \mathrm{id}_{\mathcal{C}(S_2)} \end{tikzcd} \quad \begin{tikzcd} F_{\Sigma_1 \cup \Sigma_2 \cup \Sigma_3} \arrow[r] \arrow[d] & F_{\Sigma_1} \circ F_{\Sigma_2 \cup \Sigma_3} \arrow[d] \\ F_{\Sigma_1 \cup \Sigma_2} \circ F_{\Sigma_3} \arrow[r] & F_{\Sigma_1} \circ F_{\Sigma_2} \circ F_{\Sigma_3} \end{tikzcd}$$

$$\begin{tikzcd} F_{\Sigma \sqcup \emptyset} \arrow[r] \arrow[dr] & F_\Sigma \otimes \mathrm{id}_{\mathsf{Vec}} \arrow[d] \\ & F_\Sigma \end{tikzcd} \quad \begin{tikzcd} F_{\Sigma_1 \sqcup \Sigma_2 \sqcup \Sigma_3} \arrow[r] \arrow[d] & F_{\Sigma_1} \otimes F_{\Sigma_2 \sqcup \Sigma_3} \arrow[d] \\ F_{\Sigma_1 \sqcup \Sigma_2} \otimes F_{\Sigma_3} \arrow[r] & F_{\Sigma_1} \otimes F_{\Sigma_2} \otimes F_{\Sigma_3} \end{tikzcd} \quad \begin{tikzcd} F_{(\Sigma_1 \cup \Sigma_2) \sqcup (\Sigma_1' \cup \Sigma_2')} \arrow[r] \arrow[d] & F_{\Sigma_1 \cup \Sigma_2} \otimes F_{\Sigma_1' \cup \Sigma_2'} \arrow[d] \\ F_{\Sigma_1 \sqcup \Sigma_1'} \circ F_{\Sigma_2 \sqcup \Sigma_2'} \arrow[r] & (F_{\Sigma_1} \circ F_{\Sigma_2}) \otimes (F_{\Sigma_1'} \circ F_{\Sigma_2'}) \end{tikzcd} \quad \begin{tikzcd} F_{\Sigma_1 \sqcup \Sigma_2} \arrow[r] \arrow[d] & F_{\Sigma_2} \otimes F_{\Sigma_1} \arrow[d] \\ F_{\Sigma_2 \sqcup \Sigma_1} \arrow[r] & F_{\Sigma_1} \otimes F_{\Sigma_2} \end{tikzcd}$$

(These diagrams are a bit sloppy as the functors being compared don't always have the same domain/codomain. Fixing them is not difficult, but would make them bulky.)

(2b) For every complex cobordism with thin parts Σ from S_1 to S_2 and every object $\lambda \in \mathcal{C}(S_1)$, a continuous linear map $Z_\Sigma : U(\lambda) \to U(F_\Sigma(\lambda))$.

The maps Z_Σ are natural in λ. They're also natural in Σ, meaning that for every biholomorphic map $\phi : \Sigma' \to \Sigma$ fixing S_1 and S_2 and every $\lambda \in \mathcal{C}(S_1)$, we have a commutative diagram

$$\begin{tikzcd} U(\lambda) \arrow[r, "Z_\Sigma"] \arrow[d, equal] & U(F_\Sigma(\lambda)) \arrow[d, phantom, "\cong" rotate=90] \\ U(\lambda) \arrow[r, "Z_{\Sigma'}"] & U(F_{\Sigma'}(\lambda)) \end{tikzcd}$$

We also have $Z_{\mathbf{1_S}} = \mathrm{id}_{U(\lambda)}$, $Z_{\Sigma_1 \cup \Sigma_2} = Z_{\Sigma_1} \circ Z_{\Sigma_2}$, and $Z_{\Sigma_1 \sqcup \Sigma_2} = Z_{\Sigma_1} \otimes Z_{\Sigma_2}$. (Some isomorphisms have been omitted for better readability. For example, the last equality should say that the following diagram is commutative:

$$\begin{tikzcd} U(\lambda \otimes \mu) \arrow[r, "Z_{\Sigma_1 \sqcup \Sigma_2}"] \arrow[dd, equal] & U(F_{\Sigma_1 \sqcup \Sigma_2}(\lambda \otimes \mu)) \arrow[d, phantom, "\cong" rotate=90] \\ & U(F_{\Sigma_1}(\lambda) \otimes F_{\Sigma_2}(\mu)) \arrow[d, equal] \\ U(\lambda) \otimes U(\mu) \arrow[r, "Z_{\Sigma_1} \otimes Z_{\Sigma_2}"] & U(F_{\Sigma_1}(\lambda)) \otimes U(F_{\Sigma_2}(\mu)) \end{tikzcd}$$

)

A central ext. $0 \to \mathbb{C}^\times \times \mathbb{Z} \to \widetilde{\mathrm{Ann}}(S) \to \mathrm{Ann}(S) \to 0$ of the semigroup of annuli with ∂ parametrized by S. The extension depends on the central charge $c \in \mathbb{Q}$.

(3a) For every circle S, annulus $A \in \mathrm{Ann}(S)$, and every lift $\tilde{A} \in (\mathbb{C}^\times \times \mathbb{Z}).\mathrm{Ann}_c(S)$, a trivialization $T_{\tilde{A}} : F_A(\lambda) \xrightarrow{\cong} \lambda$. *[Think: 'the map $\Sigma \mapsto F_\Sigma$ is topological.']*

These should satisfy $T_{\mathbf{1}_S} = \mathrm{id}$ and $T_{\tilde{A}_1 \cup \tilde{A}_2} = T_{\tilde{A}_1} \circ T_{\tilde{A}_2}$ (omitting the isomorphism $F_{A_1 \cup A_2} \cong F_{A_1} \circ F_{A_2}$ for better readability). Moreover, the central $\mathbb{C}^\times$ should act in the standard way: $T_{z\tilde{A}} = z \cdot T_{\tilde{A}}$ for every $z \in \mathbb{C}^\times$.

(3b) For every circle S and every object $\lambda \in \mathcal{C}(S)$, the map

$$\begin{array}{ccc} (\mathbb{C}^\times \times \mathbb{Z}).\mathrm{Ann}_c(S) & \longrightarrow & \mathrm{End}(U(\lambda)) \\ \cup\!\!\!\!\!\!\;\;\shortmid & & \cup\!\!\!\!\!\!\;\;\shortmid \\ \tilde{A} & \mapsto & \Big(U(\lambda) \xrightarrow{Z_A} U(F_A(\lambda)) \xrightarrow{U(T_{\tilde{A}})} U(\lambda)\Big) \end{array}$$

is holomorphic (holomorphic in the interior, and strong-continuous all the way to the boundary) *[Think: 'the map $A \mapsto Z_A$ is holomorphic.']*