Major Topic: Functional Analysis (Modern Analysis)

- Banach Spaces: Baire Category Theorem, Open Mapping Theorem, Closed Graph Theorem, Uniform Boundedness Principle, Hahn-Banach Theorem, Banach-Alaoglu Theorem, weak and weak-* topologies
- Topological Vector Spaces: seminorms, locally convex spaces, Krein-Milman Theorem
- Hilbert Spaces: sesquilinear forms, inner products, Cauchy-Schwartz inequality, Riesz Representation Theorem, orthonormal bases
- Operators on Hilbert Spaces: topologies on $B(H)$, polar decomposition, compact operators, trace-class operators, Hilbert-Schmidt operators, normal operators
- Examples: $C_0(X), B_2(H), L^p(X, \mu)$

References: Pederson, *Analysis Now*, Chapters 2-3

Major Topic: Operator Algebras (Modern Analysis)

- Banach Algebras: spectrum, spectral radius formula, characters, maximal ideals, Gelfand transform
- $C^*$-Algebras: adjoining an identity, spectral permanence, commutative $C^*$-algebras, continuous functional calculus, approximate identities, ideals and quotients, states, GNS construction, Gelfand-Naimark Theorem
- Von Neumann Algebras: spectral theorem for normal operators; Bicommutant Theorem; Kaplansky Density Theorem; comparison of projections in factors; definitions and examples of factors of types I, II, and III; the coupling constant; index of subfactors
- Examples: $C_0(X), B_2(H), L^\infty(X, \mu) \rtimes \alpha G \ (G$ countable and discrete)$

References: Pederson, *Analysis Now*, Chapter 4; Davidson, *$C^*$-Algebras by Example*, Chapters 1-2.2; Jones’ notes on von Neumann algebras, Chapters 2-11

Minor Topic: Algebraic Topology (Geometry and Topology)

- homotopy, fundamental group, Van Kampen’s Theorem
- fiber bundles, principal $G$-bundles, covering spaces, classification theorem for covering spaces
- CW-complexes, $\Delta$-complexes
- singular, simplicial, and cellular homology

Here are the questions (that I can remember) from my Qualifying Exam:

**Major Topic: Operator Algebras**

1. (Rieffel) What is the GNS-construction?
   Here I assumed $A$ was a $C^*$-algebra, which prompted:
   
   - (Rieffel) What if $A$ is just a normed $*$-algebra?
   - (Rieffel) What does it mean for a linear functional to be positive on a normed $*$-algebra?
   - (Rieffel) Where would you use the fact that $A$ is a $C^*$-algebra during the construction?

2. (Rieffel) Compute some examples of the GNS-construction.
   
   - (Rieffel) What about the non-commutative case? Try $M_n(\mathbb{C})$.
     I picked the trace, but Jones asked me to pick another state, so I picked a vector state.
   - (Rieffel) What else can you say about this state?
   - (Rieffel) What is a density matrix?
   - (Rieffel) Now what about computing the GNS-construction for an infinite dimensional example.
     I picked $\nu N(S_\infty)$.
   - (Rieffel) What do you know about $\nu N(S_\infty)$?
   - (Jones) What is a factor? How do you know $\nu N(S_\infty)$ is a factor?
   - (Jones) What do you get from doing the GNS-construction in this case?

**Major Topic: Functional Analysis**

1. (Teichner) Are the $L^p$ spaces locally convex?

2. (Teichner) Why do we care about local convexity?

3. (Teichner) Is $L^p$ (I interpreted as $L^p[0,1]$) isomorphic to $l^p$?

**Minor Topic: Algebraic Topology**

1. (Teichner) Can you compute as many homotopy groups as possible of $\mathbb{C}P^n$ as possible?
   I got a lot of help in this question, and it went pretty well. This question was really cool as it connected ideas in previous subjects to algebraic topology. For example, I talked about the set of rank one projections in $C^n$ in connection to $\mathbb{C}P^n$, and these projections give density matrices which correspond to pure states: If $P = |x\rangle \langle x|$, then $T \mapsto \text{tr}(PT) = \langle Tx, x \rangle$ is a pure state (and also a vector state).

2. (Ganor) Can you compute the homology of the Klein Bottle?