Math 2568 Homework 1
Math 2568 Due: August 26, 2019

Problem 1

Let $x = (2, 1, 3)$ and $y = (1, 1, -1)$ and compute the given expression.

§1.1, Exercise 2. $2x - 3y$.

$2x - 3y = (4, 2, 6) - (3, 3, -3) = (1, -1, 9)$.

Problem 2

§1.1, Exercise 4. Let A be the 3×4 matrix

$$A = \begin{pmatrix} 2 & -1 & 0 & 1 \\ 3 & 4 & -7 & 10 \\ 6 & -3 & 4 & 2 \end{pmatrix}.$$

(a) For which n is a row of A a vector in $\mathbb{R}^n$? .

(b) What is the 2^{nd} column of A?

(c) Let a_{ij} be the entry of A in the i^{th} row and the j^{th} column. What is $a_{23} - a_{31}$?

(a) The number of entries in a row is the number of columns. Thus, $n = 4$;
(b) $\begin{pmatrix} -1 \\ 4 \\ -3 \end{pmatrix}$; (c) $a_{23} - a_{31} = -7 - 6 = -13$.

Problem 3

For each of the pairs of vectors or matrices decide whether addition of the members of the pair is possible; and, if addition is possible, perform the addition.

§1.1, Exercise 7. $x = (1, 2, 3)$ and $y = (-2, 1)$.

x has three entries; y has two entries; addition is not possible.

Problem 4

Let $A = \begin{pmatrix} 2 & 1 \\ -1 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 2 \\ 3 & -1 \end{pmatrix}$ and compute the given expression.

§1.1, Exercise 10. $4A + B$.

$4A + B = \begin{pmatrix} 8 & 6 \\ -1 & 15 \end{pmatrix}$.

Problem 5 (MATLAB)

Let $x = (1.2, 1.4, -2.45)$ and $y = (-2.6, 1.1, 0.65)$ and use MATLAB to compute the given expression.

§1.2, Exercise 3.(MATLAB) $3.27x - 7.4y$.

$3.27x - 7.4y = (23.1640, -3.5620, -12.8215)$.

Problem 6 (MATLAB)

Let

$$A = \begin{pmatrix} 1.2 & 2.3 & -0.5 \\ 0.7 & -1.4 & 2.3 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -2.9 & 1.23 & 1.6 \\ -2.2 & 1.67 & 0 \end{pmatrix}$$

and use MATLAB to compute the given expression.

§1.2, Exercise 5.(MATLAB) $-4.2A + 3.1B$.

$-4.2A + 3.1B = \begin{pmatrix} -14.0300 & -5.8470 & 7.0600 \\ -9.7600 & 11.0570 & -9.6600 \end{pmatrix}$.

Problem 7

Decide whether or not the given matrix is symmetric.

§1.3, Exercise 5. $A = \begin{pmatrix} 3 & 4 & -1 \\ 4 & 3 & 1 \\ -1 & 1 & 10 \end{pmatrix}$.

Since $a_{21} = a_{12}$, $a_{31} = a_{13}$, and $a_{32} = a_{23}$, the matrix is symmetric.

Problem 8

A general 2×2 diagonal matrix has the form $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$. Thus the two unknown real numbers a and b are needed to specify each 2×2 diagonal matrix. how many unknown real numbers are needed to specify each of the given matrices:

§1.3, Exercise 11. An upper triangular 2×2 matrix?

A 2×2 upper triangular matrix A has the form $A = \begin{pmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{pmatrix}$. Thus the number of entries needed to define A is 3.

Problem 9

A general 2×2 diagonal matrix has the form $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$. Thus the two unknown real numbers a and b are needed to specify each 2×2 diagonal matrix. how many unknown real numbers are needed to specify each of the given matrices:

§1.3, Exercise 13. An $m \times n$ matrix?

Each row of the matrix has n entries and there are m rows. Hence the number of unknown entries is mn.

Problem 10

A general 2×2 diagonal matrix has the form $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$. Thus the two unknown real numbers a and b are needed to specify each 2×2 diagonal matrix. how many unknown real numbers are needed to specify each of the given matrices:

§1.3, Exercise 16. A symmetric $n \times n$ matrix?

The number of independent entries in row k of an $n \times n$ symmetric matrix is $n - k + 1$. Thus the number of independent entries in the matrix is

$$n + (n-1) + \cdots + 1 = 1 + 2 + \cdots + n = \sum_{k=1}^{n} k = \frac{n(n+1)}{2}.$$

Problem 11

Determine whether the statement is *True* or *False*?

§1.3, Exercise 18. Every diagonal matrix is a multiple of the identity matrix.

False — for example: $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$.

Problem 12

§1.4, Exercise 9. Find a real number a so that the vectors

$$x = (1, 3, 2) \quad \text{and} \quad y = (2, a, -6)$$

are perpendicular.

The vectors x and y are perpendicular when $(1, 3, 2) \cdot (2, a, -6) = 3a - 10 = 0$. Thus, $a = \frac{10}{3}$.

Problem 13 (MATLAB)

Find the angle in degrees between the given pair of vectors.

§1.4, Exercise 21.(MATLAB) $x = (2, 1, -3, 4)$ and $y = (1, 1, -5, 7)$.

$\theta = \arccos\left(\frac{x \cdot y}{||x||\,||y||}\right) = 0.2715$ radians $= 15.5570°$.