Math 2568 Homework 2

Math 2568 Due: Wednesday, September 4, 2019

Problem 1

Determine whether the given pair of vectors is perpendicular.

§1.4, Exercise 8. $x = (2, 1, 4, 5)$ and $y = (1, -4, 3, -2)$.

Answer: The vectors are perpendicular.

Solution: Compute: $(2, 1, 4, 5) \cdot (1, -4, 3, -2) = 0$.

Problem 2

§2.1, Exercise 7.

(a) Find a quadratic polynomial $p(x) = ax^2 + bx + c$ satisfying $p(0) = 1$, $p(1) = 5$, and $p(-1) = -5$.

(a) **Answer:** The quadratic $p(x) = -x^2 + 5x + 1$ satisfies these conditions.

Solution: Since $p(x) = ax^2 + bx + c$ for any quadratic equation, we find this solution by evaluating $p(0) = 1$, $p(1) = 5$, and $p(-1) = -5$, which yields the system of equations

$$\begin{array}{lcrcrcrcr} p(0) & = & & & & & c & = & 1 \\ p(1) & = & a & + & b & + & c & = & 5 \\ p(-1) & = & a & - & b & + & c & = & -5 \end{array}$$

We solve this system to obtain $(a, b, c) = (-1, 5, 1)$, then substitute these coefficients into the general quadratic.

Problem 3

§2.2, Exercise 5.

(a) Find a vector u normal to the plane $2x + 2y + z = 3$.

(b) Find a vector v normal to the plane $x + y + 2z = 4$.

(c) Find the cosine of the angle θ between the vectors u and v.

(a) $u = (2, 2, 1)$, since we know that the normal vector to the plane $ax+by+cz = d$ is (a, b, c).

(b) $v = (1, 1, 2)$.

(c) $\cos\theta = \frac{u \cdot v}{||u||\,||v||} = \frac{2}{\sqrt{6}}$. In MATLAB, type `acos(2/sqrt(6))*180/pi` to obtain $\theta = 35.2644°$.

Problem 4

Determine whether the given matrix is in reduced echelon form.

§2.3, Exercise 1. $\begin{pmatrix} 1 & -1 & 0 & 1 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & 0 \end{pmatrix}$.

The matrix is not in reduced echelon form.

Problem 5

We list the reduced echelon form of an augmented matrix of a system of linear equations. Which columns in these augmented matrices contain pivots? Describe all solutions to these systems of equations in the form of (2.3.14).

§2.3, Exercise 4. The solutions of the system are:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -4x_2 \\ x_2 \\ 5 \end{pmatrix}$$

The 1^{st} and 3^{rd} columns of the matrix contain pivots. The solutions of the system are:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -4x_2 \\ x_2 \\ 5 \end{pmatrix}$$

Problem 6

§2.3, Exercise 9. Use row reduction and back substitution to solve the following system of two equations in three unknowns:

$$\begin{array}{rcrcrcr} x_1 & - & x_2 & + & x_3 & = & 1 \\ 2x_1 & + & x_2 & - & x_3 & = & -1 \end{array}$$

Is $(1, 2, 2)$ a solution to this system? If not, is there a solution for which $x_3 = 2$?

Answer: The solution to this system is

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ x_3 - 1 \\ x_3 \end{pmatrix},$$

where x_3 is any real number.

Solution: Row reduce the augmented matrix of the system:

$$\left(\begin{array}{rrr|r} 1 & -1 & 1 & 1 \\ 2 & 1 & -1 & -1 \end{array}\right) \longrightarrow \left(\begin{array}{rrr|r} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 \end{array}\right).$$

Although $(1, 2, 2)$ is not a solution to this system, there is a solution for which $x_3 = 2$, namely $(0, 1, 2)$.

Problem 7

Determine the augmented matrix and all solutions for each system of linear equations

§2.3, Exercise 11. $\begin{array}{rcl} 2x - y + z + w & = & 1 \\ x + 2y - z + w & = & 7 \end{array}$.

The augmented matrix for this system is

$$\left(\begin{array}{rrrr|r} 2 & -1 & 1 & 1 & 1 \\ 1 & 2 & -1 & 1 & 7 \end{array}\right)$$

which can be row reduced to

$$\left(\begin{array}{rrrr|r} 1 & 0 & \frac{1}{5} & \frac{3}{5} & \frac{9}{5} \\ 0 & 1 & -\frac{3}{5} & \frac{1}{5} & \frac{13}{5} \end{array}\right).$$

The solution set is therefore

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} \frac{9}{5} - \frac{1}{5}x_3 - \frac{3}{5}x_4 \\ \frac{13}{5} + \frac{3}{5}x_3 - \frac{1}{5}x_4 \\ x_3 \\ x_4 \end{pmatrix}.$$

Problem 8

Consider the augmented matrices representing systems of linear equations, and decide

(a) if there are zero, one or infinitely many solutions, and

(b) if solutions are not unique, how many variables can be assigned arbitrary values.

§2.3, Exercise 14. $\left(\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 5 & 0 & 2 \\ 0 & 0 & 4 & 3 \end{array}\right).$

Answer: The system has a unique solution.

Solution: The row-reduced form of the matrix is:

$$\left(\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & \frac{2}{5} \\ 0 & 0 & 1 & \frac{3}{4} \end{array}\right).$$

Problem 9 (MATLAB)

Use elementary row operations and MATLAB to put each of the given matrices into row echelon form. Suppose that the matrix is the augmented matrix for a system of linear equations. Is the system consistent or inconsistent?

§2.3, Exercise 24.(MATLAB)

$$\begin{pmatrix} -2 & 1 & 9 & 1 \\ 3 & 3 & -4 & 2 \\ 1 & 4 & 5 & 5 \end{pmatrix}.$$

The row-reduced matrix is:

```
A =
    1.0000   -0.5000   -4.5000   -0.5000
         0    1.0000    2.1111    0.7778
         0         0         0    2.0000
```

This matrix represents an inconsistent linear system.

Problem 10

§2.4, Exercise 3. The augmented matrix of a consistent system of five equations in seven unknowns has rank equal to three. How many parameters are needed to specify all solutions?

Answer: Four parameters are needed to specify all solutions.

Solution: According to Theorem 2.4.6, $n - \ell$ parameters are needed to parameterize the set of all solutions of a linear system, where n is the number of unknowns, and ℓ is the rank of the reduced echelon matrix. In this case, $n = 7$ and $\ell = 3$.

Problem 11 (MATLAB)

Use `rref` on the given augmented matrices to determine whether the associated system of linear equations is consistent or inconsistent. If the equations are consistent, then determine how many parameters are needed to enumerate all solutions.

§2.4, Exercise 5.(MATLAB)

$$A = \left(\begin{array}{rrrrr|r} 2 & 1 & 3 & -2 & 4 & 1 \\ 5 & 12 & -1 & 3 & 5 & 1 \\ -4 & -21 & 11 & -12 & 2 & 1 \\ 23 & 59 & -8 & 17 & 21 & 4 \end{array} \right) \tag{1}$$

Answer: Matrix A is consistent and requires 3 parameters to enumerate all solutions.

Solution:

```
rref(A) =
    1.0000         0    1.9474   -1.4211    2.2632    0.5789
         0    1.0000   -0.8947    0.8421   -0.5263   -0.1579
         0         0         0         0         0         0
         0         0         0         0         0         0
```

Problem 12 (MATLAB)

Compute the rank of the given matrix.

§2.4, Exercise 10.(MATLAB) $\begin{pmatrix} 2 & 1 & 0 & 1 \\ -1 & 3 & 2 & 4 \\ 5 & -1 & 2 & -2 \end{pmatrix}$.

The rank of the matrix is 3.