Math 2568 Homework 3

Math 2568 Due: Monday, September 9, 2019

Problem 1

Row reduce the given matrix to reduced echelon form by hand and determine its rank.

§2.4, Exercise 1. $A = \begin{pmatrix} 1 & 2 & 1 & 6 \\ 3 & 6 & 1 & 14 \\ 1 & 2 & 2 & 8 \end{pmatrix}$

Problem 2

§2.4, Exercise 3.

How many solutions does the equation

$$A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$$

have for the following choices of A. Explain your reasoning.

(a) $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

(b) $A = \begin{pmatrix} 1 & 3 & 1 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(c) $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

Problem 3

§2.4, Exercise 6.

Consider the system of equations

$$\begin{aligned} x_1 + 3x_3 &= 1 \\ -x_1 + 2x_2 - 3x_3 &= 1 \\ 2x_2 + ax_3 &= b \end{aligned}$$

For which real numbers a and b does the system have no solutions, a unique solution, or infinitely many solutions? Your answer should subdivide the ab-plane into three disjoint sets.

Problem 4

§2.4, Exercise 14. Prove that the rank of an $m \times n$ matrix A is less than or equal to the minimum of m and n.

Problem 5

§3.1, Exercise 1. Let

$$A = \begin{pmatrix} 2 & 1 \\ -1 & 4 \end{pmatrix} \quad \text{and} \quad x = \begin{pmatrix} 3 \\ -2 \end{pmatrix}.$$

Compute Ax.

Problem 6

§3.1, Exercise 7. Let

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \quad \text{and} \quad x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}.$$

Denote the columns of the matrix A by

$$A_1 = \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix}, \quad A_2 = \begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix}, \quad \cdots \quad A_n = \begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix}.$$

Show that the matrix vector product Ax can be written as

$$Ax = x_1 A_1 + x_2 A_2 + \cdots + x_n A_n,$$

where $x_j A_j$ denotes scalar multiplication (see Chapter 1).

Problem 7

§3.1, Exercise 9. Write the system of linear equations

$$\begin{aligned} 2x_1 + 3x_2 - 2x_3 &= 4 \\ 6x_1 - 5x_3 &= 1 \end{aligned}$$

in the matrix form $Ax = b$.

Problem 8

§3.1, Exercise 10. Find all solutions to

$$\begin{pmatrix} 1 & 3 & -1 & 4 \\ 2 & 1 & 5 & 7 \\ 3 & 4 & 4 & 11 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 14 \\ 17 \\ 31 \end{pmatrix}.$$

Problem 9

§3.1, Exercise 13. Is there an upper triangular 2×2 matrix A such that

$$A \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}? \tag{1}$$

Is there a symmetric 2×2 matrix A satisfying (1)?