Math 2568 Homework 4

Math 2568 Due: Monday, September 16, 2019

Problem 1

§2.4, Exercise 15. Consider the augmented matrix

$$A = \left(\begin{array}{cc|c} 1 & -r & 1 \\ r & -1 & 1 \end{array} \right)$$

where r is a real parameter.

1 Find all r so that $\text{rank}(A) = 2$.

2 Find all r for which the corresponding linear system has

(a) no solution,

(b) one solution, and

(c) infinitely many solutions.

Problem 2

§3.1, Exercise 12. Let A be a 2×2 matrix. Find A so that

$$A \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$
$$A \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}.$$

Problem 3

Determine whether the given transformation is linear.

§3.3, Exercise 7. $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x_1, x_2) = (x_1 + x_1 x_2, 2x_2)$.

Problem 4

Determine whether the given transformation is linear.

§3.3, Exercise 9. $T : \mathbb{R}^2 \to \mathbb{R}^3$ defined by $T(x_1, x_2) = (1, x_1 + x_2, 2x_2)$

Problem 5

§3.3, Exercise 14. Let σ permute coordinates cyclically in $\mathbb{R}^3$; that is,

$$\sigma(x_1, x_2, x_3) = (x_2, x_3, x_1).$$

Find a 3×3 matrix A such that $\sigma = L_A$.

Problem 6

§3.3, Exercise 16. Let $P : \mathbb{R}^n \to \mathbb{R}^m$ and $Q : \mathbb{R}^n \to \mathbb{R}^m$ be linear mappings. Prove that $S : \mathbb{R}^n \to \mathbb{R}^m$ defined by

$$S(x) = P(x) + Q(x)$$

is also a linear mapping. Theorem 3.3.5 states that there are matrices A, B and C such that

$$P = L_A \quad \text{and} \quad Q = L_B \quad \text{and} \quad S = L_C.$$

What is the relationship between the matrices A, B, and C?

Problem 7

§3.4, Exercise 3.

(a) Find all solutions to the homogeneous equation $Ax = 0$ where

$$A = \begin{pmatrix} 2 & 3 & 1 \\ 1 & 1 & 4 \end{pmatrix}.$$

(b) Find a single solution to the inhomogeneous equation

$$Ax = \begin{pmatrix} 6 \\ 6 \end{pmatrix}. \tag{1}$$

(c) Use your answers in (a) and (b) to find all solutions to (1).

Problem 8

§3.4, Exercise 4. How many solutions can a homogeneous system of 4 linear equations in 7 unknowns have?

Problem 9

Compute the given matrix product.

§3.5, Exercise 8. $\begin{pmatrix} 2 & -1 & 3 \\ 1 & 0 & 5 \\ 1 & 5 & -1 \end{pmatrix} \begin{pmatrix} 1 & 7 \\ -2 & -1 \\ -5 & 3 \end{pmatrix}.$

Problem 10

§3.6, Exercise 4. Let

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{and} \quad J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$

(a) Show that $J^2 = -I$.

(b) Evaluate $(aI + bJ)(cI + dJ)$ in terms of I and J.

Problem 11

§3.7, Exercise 1. Verify by matrix multiplication that the following matrices are inverses of each other:

$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} -1 & 0 & 2 \\ 2 & -1 & -2 \\ 1 & 0 & -1 \end{pmatrix}.$$

Problem 12 (MATLAB)

§3.6, Exercise 10 (MATLAB).(MATLAB) Experimentally, find two symmetric 2×2 matrices A and B for which the matrix product AB is *not* symmetric.