Math 2568 Homework 5
Math 2568 Due: Monday, September 23, 2019

Problem 1

§3.4, Exercise 2. Write all solutions to the homogeneous system of linear equations

$$\begin{aligned} x_1 + 2x_2 + x_4 - x_5 &= 0 \\ x_3 - 2x_4 + x_5 &= 0 \end{aligned}$$

as the general superposition of three vectors.

Problem 2

§3.3, Exercise 12. The *cross product* of two 3-vectors $x = (x_1, x_2, x_3)$ and $y = (y_1, y_2, y_3)$ is the 3-vector

$$x \times y = (x_2y_3 - x_3y_2, -(x_1y_3 - x_3y_1), x_1y_2 - x_2y_1).$$

Let $K = (2, 1, -1)$.

(a) Show that the mapping $L : \mathbb{R}^3 \to \mathbb{R}^3$ defined by

$$L(x) = x \times K$$

is a linear mapping.

(b) Find the 3×3 matrix A such that

$$L(x) = Ax,$$

that is, $L = L_A$.

Problem 3

Determine whether or not the matrix products AB or BA can be computed for each given pair of matrices A and B. If the product is possible, perform the computation.

§3.5, Exercise 2. $A = \begin{pmatrix} 0 & -2 & 1 \\ 4 & 10 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 2 \\ 3 & -1 \end{pmatrix}$.

Problem 4

§3.7, Exercise 2. Let $\alpha \neq 0$ be a real number and let A be an invertible matrix. Show that the inverse of the matrix αA is given by $\frac{1}{\alpha}A^{-1}$.

Problem 5

Use row reduction to find the inverse of the given matrix.

§3.7, Exercise 5. $\begin{pmatrix} 1 & 4 & 5 \\ 0 & 1 & -1 \\ -2 & 0 & -8 \end{pmatrix}$.

Problem 6

§3.7, Exercise 10. For which values of a, b, c is the matrix

$$A = \begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix}$$

invertible? Find A^{-1} when it exists.

Problem 7

§3.7, Exercise 14. Let A and B be 3×3 invertible matrices so that

$$A^{-1} = \begin{pmatrix} 1 & 0 & -1 \\ -1 & -1 & 0 \\ 0 & 1 & -1 \end{pmatrix} \quad \text{and} \quad B^{-1} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Without computing A or B, determine the following:

(a) $\text{rank}(A)$

(b) The solution to

$$Bx = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

(c) $(2BA)^{-1}$

Problem 8

§3.8, Exercise 4. Let A be a 2×2 matrix having integer entries. Find a condition on the entries of A that guarantees that A^{-1} has integer entries.

Problem 9

§3.7, Exercise 15. True or False: Determine whether the following statements are true or false, and explain your answer.

(a) The only 3×2 matrix A so that $Ax = 0$ for all $x \in \mathbb{R}^2$ is $A = 0$.

(b) A system of 5 equations in 3 unknowns with the solution $x_1 = 0, x_2 = -3, x_3 = 1$ must have infinitely many solutions.