Homework 10
Math 2568

Problem 1

In Exercises 15-20 decide whether the statement is true or false, and explain your answer.

§5.6, Exercise 16. Every set of four vectors in $\mathbb{R}^3$ is linearly dependent.

Problem 2

In Exercises 15-20 decide whether the statement is true or false, and explain your answer.

§5.6, Exercise 20. If U is a subspace of $\mathbb{R}^3$ of dimension 1 and V is a subspace of $\mathbb{R}^3$ of dimension 2, then $U \cap V = \{0\}$.

Problem 3

Consider the matrix

$$C = \begin{pmatrix} -1 & -10 & -6 \\ 0 & 4 & 3 \\ 0 & -14 & -9 \end{pmatrix}.$$

§6.1, Exercise 15. Verify that

$$v_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad v_2 = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \qquad \text{and} \qquad v_3 = \begin{pmatrix} 6 \\ -3 \\ 7 \end{pmatrix}$$

are eigenvectors of C and find the associated eigenvalues.

Problem 4

Compute the general solution for the given system of differential equations.

§6.2, Exercise 6. $\dfrac{dX}{dt} = \begin{pmatrix} 5 & -1 \\ 1 & 3 \end{pmatrix} X.$

Problem 5

Compute the general solution for the given system of differential equations.

§6.2, Exercise 7. $\dfrac{dX}{dt} = \begin{pmatrix} -4 & 4 \\ -1 & 0 \end{pmatrix} X.$

Problem 6

§6.3, Exercise 1. Suppose that the matrices A and B are similar and the matrices B and C are similar. Show that A and C are also similar matrices.

Problem 7

Determine whether or not the given matrices are similar, and why.

§6.3, Exercise 4. $C = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}$ and $D = \begin{pmatrix} 4 & -2 \\ -2 & 4 \end{pmatrix}$.

Problem 8

§6.3, Exercise 5. Let $B = P^{-1}AP$ so that A and B are similar matrices. Suppose that v is an eigenvector of B with eigenvalue λ. Show that Pv is an eigenvector of A with eigenvalue λ.

Problem 9

§6.3, Exercise 7. Solve the initial value problem

$$\begin{aligned} \dot{x} &= 2x + 3y \\ \dot{y} &= -3x + 2y \end{aligned}$$

where $x(0) = 1$ and $y(0) = -2$.

Problem 10

§6.3, Exercise 8. Solve the initial value problem

$$\begin{aligned} \dot{x} &= -2x + y \\ \dot{y} &= -2y \end{aligned}$$

where $x(0) = 4 \quad \text{and} \quad y(0) = -1$.