Homework 7
Math 2568 Feb 27, 2019

Problem 1

Find the solution to the system of differential equations $\dot{X} = CX$ satisfying $X(0) = X_0$.

§4.7, Exercise 2. $C = \begin{pmatrix} 2 & -3 \\ 0 & -1 \end{pmatrix}$ and $X_0 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$.

Problem 2

§4.7, Exercise 5. Solve the initial value problem $\dot{X} = CX$ where $X_0 = e_1$ given that

(a) $X(t) = e^{-t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ is a solution,

(b) $\mathrm{tr}(C) = 3$, and

(c) C is a symmetric matrix.

Problem 3 (MATLAB)

With MATLAB assistance, find the solution to the system of differential equations $\dot{X} = CX$ satisfying $X(0) = X_0$.

§4.7, Exercise 6.(MATLAB) $C = \begin{pmatrix} 1.76 & 4.65 \\ 0.23 & 1.11 \end{pmatrix}$ and $X_0 = \begin{pmatrix} 0.34 \\ -0.50 \end{pmatrix}$.

Problem 4 (MATLAB)

Find the solution to $\dot{X} = CX$ satisfying $X(0) = X_0$ in two different ways, as follows.

(a) Use pplane9 to find $X(0.5)$. **Hint**: Use the Specify a computation interval option in the PPLANE9 Keyboard input window to compute the solution

to $t = 0.5$. Then use the zoom in square feature to determine an answer to three decimal places.

(b) Next use MATLAB to find the eigenvalues and eigenvectors of C and to find a closed form solution $X(t)$. Use this formula to evaluate $X(0.5)$ to three decimal places.

(c) Do the two answers agree?

§4.7, Exercise 8.(MATLAB) $C = \begin{pmatrix} 2.65 & -2.34 \\ -1.5 & -1.2 \end{pmatrix}$ and $X_0 = \begin{pmatrix} 0.5 \\ 0.1 \end{pmatrix}$.

Problem 5

§5.1, Exercise 3. Let

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & -1 & 1 \end{pmatrix}.$$

Let V_3 be the set of vectors $x \in \mathbb{R}^3$ such that $Ax = 0$. Verify that V_3 is a subspace of $\mathbb{R}^3$. Compare V_1 with V_3.