Weeds and classification

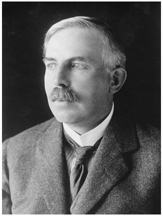
David Penneys, Mathematics

STEAM Exchange

4/20/2017

A controversial quote

"All science is either physics or stamp collecting."



Ernest Rutherford, 1871-1937

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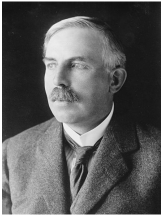

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- Taxonomy/classification is an important theme in science.

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- Interesting examples may arise through *classification*.

Gardening: a metaphor for classification

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Math fun fact

The number of petals on a flower is typically a Fibonacci number!
1,1,2,3,5,8,13,21,34...

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Example: shapes

Shapes	number of sides
triangle	3
quadrilateral	4
$\vdots$	$\vdots$
heptadecagon	17
$\vdots$	$\vdots$

Can we find *all* the objects?

Find all the objects!

[Hyperbole and a half]

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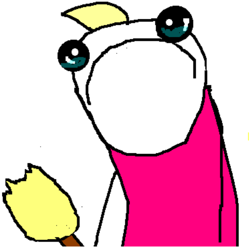

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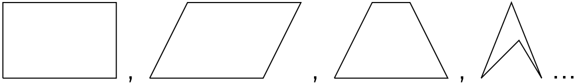

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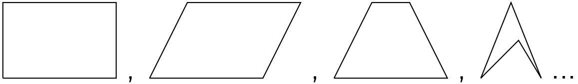

Technique

- Look at all objects with the same invariant.

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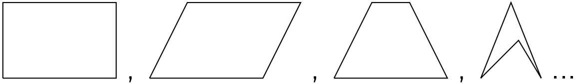

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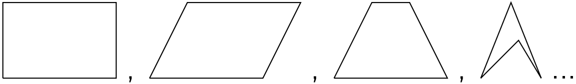

Technique

- Look at all objects with the same invariant.
- We'll get everything we want, but we may get *extra stuff* we don't want.
- We remove these *weeds* by hand from our classification.

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I study *fusion categories*, which are mathematical models of systems of *elementary particles* together with *fusion rules* which tells us how they merge and split.

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- Tea and cookies!

Life goal: find interesting fusion categories

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- Particles $\{\circ, \bullet\}$, fusion rule $\begin{array}{c|cc} \otimes & \circ & \bullet \\ \hline \circ & \circ & \bullet \\ \bullet & \bullet & \circ \end{array}$ (adding even/odd #s)

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- Particles $\{\circ, {\color{blue}\bullet}\}$, fusion rule $\begin{array}{c|cc} \otimes & \circ & {\color{blue}\bullet} \\ \hline \circ & \circ & {\color{blue}\bullet} \\ {\color{blue}\bullet} & {\color{blue}\bullet} & \circ \end{array}$ $\left(\begin{array}{c|cc} + & 2 & 3 \\ \hline 2 & 4 & 5 \\ 3 & 5 & 6 \end{array}\right)$

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While it looks like any fusion rule works, there are many difficult constraints they must satisfy.

Classifying small fusion categories

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Possible fusion rules for 2 particle systems:

Indexed by whole numbers $n = 0, 1, 2, 3, \ldots$:
$$\begin{array}{c|cc} \otimes & \circ & \color{red}{\bullet} \\ \hline \circ & \circ & \color{red}{\bullet} \\ \color{red}{\bullet} & \color{red}{\bullet} & \circ + n\,\color{red}{\bullet} \end{array}$$

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- Only the cases $n = 0$ or 1 are genuine fusion rules.
- The cases $n \geq 2$ are *weeds* which must be removed *by hand*.

The golden fusion category

The golden fusion category has 2 elementary particles and $n = 1$:

Particles $\{\circ, \bullet\}$, fusion rule

$\otimes$	$\circ$	$\bullet$
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Let's see what happens when we keep smashing $\textcolor{orange}{\bullet}$'s together:

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$$ {\color{orange}\bullet} \otimes {\color{orange}\bullet} = \circ + {\color{orange}\bullet} $$

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$$\begin{aligned}
\bullet \otimes \bullet \otimes \bullet &= \bullet \otimes (\bullet \otimes \bullet) && \text{(associate)} \\
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&= \bullet + 2(\circ + \bullet) \\
&= 2\circ + 3\,\bullet
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$\#\bullet$	result
1	$0\circ + 1\bullet$
2	$1\circ + 1\bullet$
3	$1\circ + 2\bullet$
4	$2\circ + 3\bullet$
5	$3\circ + 5\bullet$
6	$5\circ + 8\bullet$
7	$8\circ + 13\bullet$
$\vdots$	$\vdots$

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The coefficients are the Fibonacci numbers!

Thank you for listening!

Slides available at:

https://people.math.osu.edu/penneys.2/PenneysSTEAM2017.pdf