NAME : KEY

OSU Name.# : ____________

Lecturer:: ____________

Recitation Instructor : ____________

Recitation Time : ____________

INSTRUCTIONS

- **SHOW ALL WORK** in problems 2, 3, and 5.
  Incorrect answers with work shown may receive partial credit, but unsubstantiated correct answers may receive NO credit.

  You don't have to show work in problems 1 and 4.

- Give EXACT answers unless asked to do otherwise.

- You do not need to simplify numerical answers such as $\frac{5}{\sqrt{8}} - \frac{3}{\sqrt{32}}$.

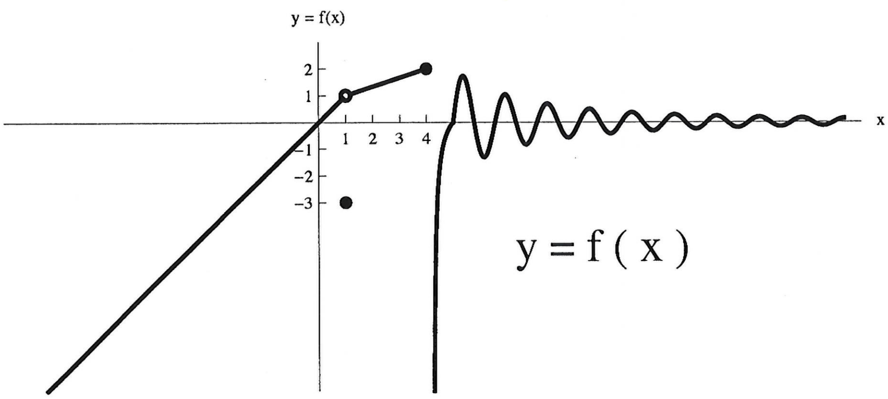
- Calculators are permitted EXCEPT those calculators that have computer algebra systems (CAS) or ability to communicate with others. Furthermore, all memory must be cleared and all apps must be removed. PDA's, laptops, and cell phones are prohibited. Do not have these devices out!

- The exam duration is 55 minutes.

- The exam consists of 5 problems starting on page 2 and ending on page 8. Make sure your exam is not missing any pages before you start.

PROBLEM NUMBER	SCORE
1	(32)
2	(16)
3	(24)
4	(16)
5	(12)
TOTAL	(100)

1. (32 pts)
The graph of a function f is given in the figure below.
The domain of f is $(-\infty, +\infty)$.
Use the graph of f to answer the questions below.



(I) (2 pts) Find the range of f.

Range of f = $(-\infty, 2]$

(II) Find the following values.

(Note : Possible answers include $+\infty$, $-\infty$, or "does not exist".)

(a) (2 pts) $\lim_{x \to 1^+} f(x) =$ 1

(b) (2 pts) $\lim_{x \to 1^-} f(x) =$ 1

(c) (2 pts) $\lim_{x \to 1} f(x) =$ 1

(d) (2 pts) $f(1) =$ -3

1. (CONTINUED)

(e) (2 pts) $\lim_{x \to 4^-} f(x) =$ 2

(f) (2 pts) $\lim_{x \to 4^+} f(x) =$ $-\infty$

(g) (2 pts) $\lim_{x \to 4} f(x) =$ Does not exist

(h) (2 pts) $\lim_{x \to -\infty} f(x) =$ $-\infty$

(i) (2 pts) $\lim_{x \to +\infty} f(x) =$ 0

(j) (2 pts) $f'(0) =$ 1

(III) (2 pts) Find all vertical asymptotes.

$x = 4$

(IV) (2 pts) Find all horizontal asymptotes.

$y = 0$

(V) (2 pts) Find all slant asymptotes.

$y = x$

(VI) (4 pts) Determine the intervals of continuity for f .

$(-\infty, 1), (1, 4], (4, \infty)$

2. (16 pts) Evaluate the limit. Show your work.
(Note: Possible answers include $+\infty$ or $-\infty$.)

You may NOT use a table of values, a graph, or L'Hospitals's Rule to justify your answer.

$\boxed{\dfrac{25-20-5}{0} = \dfrac{0}{0} \text{ more algebra}}$

(a) $\displaystyle\lim_{x\to 5^+} \frac{x^2 - 4x - 5}{|x - 5|} = \lim_{x\to 5^+} \frac{x^2-4x-5}{x-5} = \lim_{x\to 5} \frac{(x-5)(x+1)}{x-5}$

For $x>5$, $|x-5| = (x-5)$

$= \displaystyle\lim_{x\to 5} x+1 = 6$

$\boxed{\dfrac{\sqrt{1}-1}{5-5} = \dfrac{0}{0} \text{ more algebra}}$

(b) $\displaystyle\lim_{x\to 5} \frac{\sqrt{x-4} - 1}{x - 5} = \lim_{x\to 5} \frac{\sqrt{x-4}-1}{x-5} \cdot \frac{\sqrt{x-4}+1}{\sqrt{x-4}+1}$

$= \displaystyle\lim_{x\to 5} \frac{(x-4)-1^2}{(x-5)(\sqrt{x-4}+1)}$

$= \displaystyle\lim_{x\to 5} \frac{x-5}{(x-5)(\sqrt{x-4}+1)}$

$= \displaystyle\lim_{x\to 5} \frac{1}{\sqrt{x-4}+1} = \frac{1}{\sqrt{5-4}+1} = \frac{1}{2}$

3. (24 pts) Let $g(x) = \begin{cases} -2e^x & \text{if } x < 0 \\ \dfrac{x+6}{x-3} & \text{if } x \geq 0. \end{cases}$

(I) Evaluate the limit. (Note: Possible answers include $+\infty$ or $-\infty$.) [You may not use a table of values, a graph, or L'Hospitals's Rule to justify your answer.] SHOW YOUR WORK.

(a) (3 pts) $\lim_{x\to 0^-} g(x) = \lim_{x\to 0^-} -2e^0 = -2\cdot 1 = -2$

(b) (3 pts) $\lim_{x\to 0^+} g(x) = \lim_{x\to 0^+} \frac{x+6}{x-3} = \frac{6}{-3} = -2$

(c) (3 pts) $\lim_{x\to 3^-} g(x) = \lim_{x\to 3^-} \frac{x+6 \to 9}{x-3 \to 0^-}$ $\boxed{= -\infty}$

(d) (3 pts) $\lim_{x\to 3^+} g(x) = \lim_{x\to 3^+} \frac{x+6 \to 9}{x-3 \to 0^+} = +\infty$

(e) (3 pts) $\lim_{x\to -\infty} g(x) = \lim_{x\to -\infty} -2e^x = -2\cdot 0 = 0$

(f) (3 pts) $\lim_{x\to +\infty} g(x) = \lim_{x\to\infty} \frac{x+6}{x-3}\cdot\frac{1/x}{1/x} = \lim_{x\to\infty} \frac{1+\frac{6}{x}}{1-\frac{3}{x}} = \frac{1+0}{1-0} = 1$

(II) (6 pts) Using the definition of continuity, show that g is continuous at 0. SHOW YOUR WORK.

$\lim_{x\to 0^-} g(x) = -2 = \lim_{x\to 0^+} g(x) \Rightarrow \lim_{x\to 0} g(x) = -2$ exists

$g(0) = \frac{0+6}{0-3} = -2$

and Since $\lim_{x\to 0} g(x) = g(0)$, g is continuous at 0.

4. (16 pts) MULTIPLE CHOICE ! CIRCLE THE CORRECT ANSWER IN EACH PART.

(I) (4 pts)

Complete the statement of the Intermediate Value Theorem :

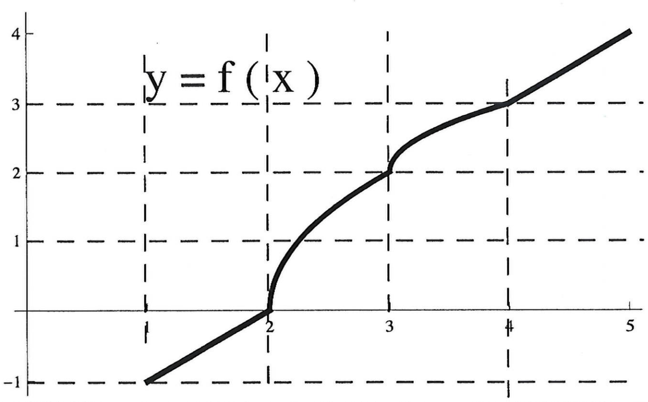
Suppose f is continuous on the interval [a, b] and L is a number between f (a) and f (b). Then there is at least one number c in (a, b) such that

(a) $c = L$; (b) $f(c) = 0$; (c) $f(c) = L$; (d) $f'(c) = L$;

(e) $c = 0$; (f) $f(a) < c < f(b)$; (g) NONE OF THE PREVIOUS ANSWERS.

(II) (4 pts)

The figure shows the graph of a function f.
At what point (points) c does the conclusion of the Intermediate Value Theorem hold for f (x) on the interval [1, 5] and L = 2.



(a) $c = 1$; (b) $c = 0$; (c) $c = 2$; (d) $c = 3$;

(e) $c = 4$; (f) $c = 5$; (g) $c = -1$.

4. MULTIPLE CHOICE ! CIRCLE THE CORRECT ANSWER IN EACH PART.

(III) (4 pts)

Given that

$$\cos x \le f(x) \le e^x \quad , \quad \text{for all } x > 0$$

evaluate $\lim_{x \to 0^+} f(x)$

$\lim_{x\to 0^+} \cos x = \cos(0) = 1$

$\lim_{x\to 0^+} e^x = e^0 = 1$

by squeeze theorem

(a) $\lim_{x\to 0^+} f(x) = 0$;

$\boxed{\text{(b) } \lim_{x\to 0^+} f(x) = 1\text{;}}$

(c) $\lim_{x\to 0^+} f(x) = e$;

(d) $\lim_{x\to 0^+} f(x)$ DOES NOT EXIST ;

(g) WE DON'T HAVE ENOUGH INFORMATION TO ANSWER THE QUESTION

(IV) (4 pts)

Given the function $f(x) = \dfrac{1}{x-2}$, find its inverse, $f^{-1}(x)$

$y = \frac{1}{x-2} \rightarrow x-2 = \frac{1}{y} \rightarrow x = 2 + \frac{1}{y}$

$\Rightarrow f^{-1}(x) = 2 + \frac{1}{x}$

(a) $f^{-1}(x) = x - 2$;

(b) $f^{-1}(x) = x + 2$;

(c) $f^{-1}(x) = \dfrac{1}{x} + 2$;

(f) $f^{-1}(x) = \dfrac{1}{x} - 2$;

(e) $f^{-1}(x)$ DOES NOT EXIST .

The slope of the line tangent to the curve $y = f(x)$

at the point $P(a, f(a))$ is given by

$$m_{tan} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}, \text{ if the limit exists.}$$

5. (12 pts) Let $f(x) = \dfrac{1}{x}$ and $a = 5$.

(a) Using the definition above, find the slope, m_{tan}, of the line tangent to the graph of f at $P(a, f(a))$.

$$m_{tan} = f'(5) = \lim_{h \to 0} \frac{f(5+h) - f(5)}{h} = \lim_{h \to 0} \frac{\frac{1}{5+h} - \frac{1}{5}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{5}{5(5+h)} - \frac{(5+h)}{5(5+h)}}{h} = \lim_{h \to 0} \frac{5 - (5+h)}{h \cdot 5 \cdot (5+h)}$$

$$= \lim_{h \to 0} \frac{-h}{h \cdot 5 \cdot (5+h)} = \lim_{h \to 0} \frac{-1}{5(5+h)} = \frac{-1}{5 \cdot 5} = \frac{-1}{25}$$

$$m_{tan} = \frac{-1}{25}$$

(b) Find an equation of the tangent line in part (a).

$$f(5) = \frac{1}{5}$$

$$\boxed{y - \frac{1}{5} = \frac{-1}{25}(x - 5)}$$