KEY

Math 1151
MIDTERM 2
March 5, 2014
Form C

NAME : ___

OSU Name.# : ___

Lecturer:: ___

Recitation Instructor : ___

Recitation Time : ___

INSTRUCTIONS

- SHOW ALL WORK in problems 1, 2, and 3.
  Incorrect answers with work shown may receive partial credit, but unsubstantiated correct answers may receive NO credit.

  You don't have to show work in problems 4 and 5.
- Give EXACT answers unless asked to do otherwise.
- You do not need to simplify numerical answers such as $\frac{5}{\sqrt{8}} - \frac{3}{\sqrt{32}}$.
- Calculators are permitted EXCEPT those calculators that have computer algebra systems (CAS) or ability to communicate with others. Furthermore, all memory must be cleared and all apps must be removed. PDA's, laptops, and cell phones are prohibited. Do not have these devices out!
- The exam duration is 55 minutes.
- The exam consists of 5 problems starting on page 2 and ending on page 8. Make sure your exam is not missing any pages before you start.

PROBLEM NUMBER	SCORE
1	(24)
2	(20)
3	(20)
4	(18)
5	(18)
TOTAL	(100)

1. (24 pts) Show your work!

(I) Evaluate the derivatives of the following functions.

NO NEED TO SIMLIFY!

(a) (8 pts) $y = \sqrt{\tan(x^5 + 4)}$

$$y' = \frac{1}{2\sqrt{\tan(x^5+4)}} \cdot \boxed{\tan(x^5+4)}'$$

$$y' = \frac{1}{2\sqrt{\tan(x^5+4)}} \cdot \sec^2(x^5+4) \cdot \boxed{x^5+4}'$$

$$y' = \frac{1}{2\sqrt{\tan(x^5+4)}} \cdot \sec^2(x^5+4) \cdot (5x^4+0)$$

(b) (8 pts) $y = x^{\cos x}$

There is 2 ways to do this.

$y = x^{\cos x} = (e^{\ln x})^{\cos x}$	$\ln y = \ln(x^{\cos x}) = \cos x \cdot \ln x$
$y = e^{\cos x \cdot \ln x}$	$\frac{d}{dx} \ln y = \frac{d}{dx}[\cos x \cdot \ln x]$
$y' = e^{\cos x \cdot \ln x} \cdot [\cos x \cdot \ln x]'$	$\frac{y'}{y} = -\sin x \cdot \ln x + \cos x \cdot \frac{1}{x}$
$\Rightarrow y' = e^{\cos x \cdot \ln x} \cdot [-\sin x \cdot \ln x + \cos x \cdot \frac{1}{x}]$	$y' = y(-\sin x \cdot \ln x + \cos x \cdot \frac{1}{x})$
	$y' = x^{\cos x}(-\sin x \cdot \ln x + \cos x \cdot \frac{1}{x})$

(II) (8 pts) Use <u>implicit</u> differentiation to find $\frac{dy}{dx}$.

$$x^2 - y^2 = 4$$

$$\frac{d}{dx}\left[x^2 - y^2\right] = \frac{d}{dx}4$$

$$2x - \frac{d}{dx}[y^2] = 0$$

$$2x - 2y \cdot \frac{dy}{dx} = 0$$

$$-2y \cdot \frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{-2y} = \frac{x}{y}$$

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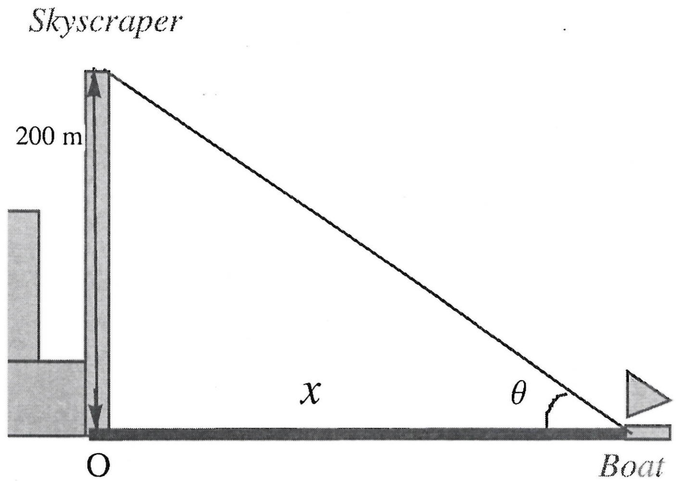
2. (20 pts)

A boat sails directly toward a 200 - meter skyscraper that stands on the edge of a harbour. The angular size θ of the building is the angle formed by lines from the top and bottom of the building to the observer (see figure).



$\tan\theta = \frac{200}{x}$

(a) Express the angle θ as the function of x, the distance of the boat from the building.

$\theta(x) = \tan^{-1}\left(\frac{200}{x}\right)$ or $\arctan\left(\frac{200}{x}\right)$

there are other answers but they are more complicated.

(b) The boat is sailing directly toward the skyscraper at 3 m / s.

Find $\frac{d\theta}{dt}$ when the boat is $x = 500$ m from the building.

$$\frac{d\theta}{dt} = \frac{1}{1+\left(\frac{200}{x}\right)^2} \cdot \frac{d}{dt}\left[\frac{200}{x}\right] = \frac{1}{1+\left(\frac{200}{x}\right)^2} \cdot \frac{-200}{x^2} \cdot \frac{dx}{dt}$$

$x = 500\text{m}, \quad \frac{dx}{dt} = -3\frac{\text{m}}{\text{s}}$

$\boxed{x \text{ is shrinking!}}$

$$\frac{d\theta}{dt} = \frac{1}{1+\left(\frac{200}{500}\right)^2} \cdot \frac{-200}{500^2} \cdot (-3) = \frac{1}{1+\left(\frac{2}{5}\right)^2} \cdot \frac{6}{5 \cdot 500} \frac{\text{rad}}{\text{s}}$$

Pay attention to signs.

[Note that it makes sense that θ increases!]

3. (20 pts)

Sketch (neatly!) the graph of a function f satisfying all of the following conditions:

(a) $f(0) = 10$, $f(-6) = 10$

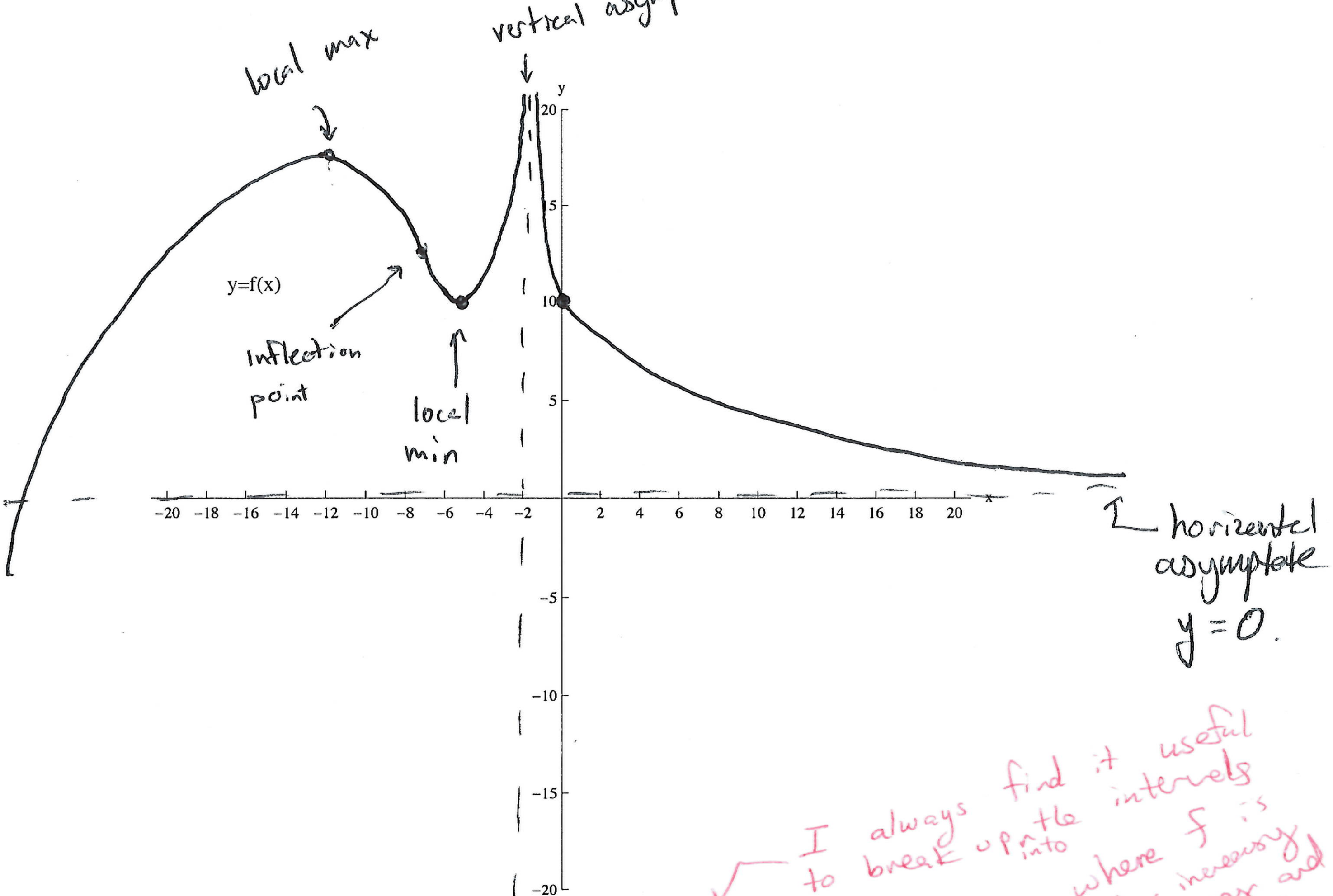
(b) $\lim_{x\to -2} f(x) = +\infty$, $\lim_{x\to +\infty} f(x) = 0$,

(c) $f'(x) > 0$ on $(-\infty, -12)$, and $(-6, -2)$,

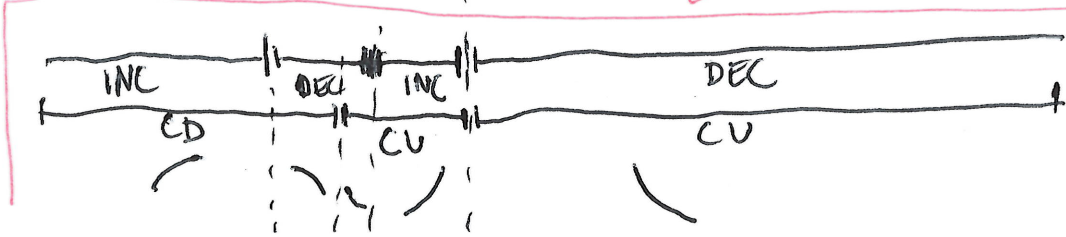
(d) $f'(x) < 0$ on $(-12, -6)$, and $(-2, +\infty)$,

(e) $f''(x) > 0$ on $(-8, -2)$, and $(-2, +\infty)$,

(f) $f''(x) < 0$ on $(-\infty, -8)$.



I always find it useful to break up the into intervals where f is either increasing or decreasing and either concave up or down.



Notice that we are given $f'(x)$, not $f(x)$.
IMPORTANT!

4. (18 pts) MULTIPLE CHOICE ! ! !

The derivative of function f is given and a table of values for

g (x) and g ' (x) is shown below.

Suppose that g is a one - to - one function and $g^{-1}(x)$ is its inverse.

$$f'(x) = \frac{x}{x^2 + 1} \quad ,$$

x	g (x)	g ' (x)
1	2	4
2	3	5
3	4	1

CIRCLE ALL THE CORRECT ANSWERS IN EACH PART.

$f'(x)=0$ when top $=0$, $\rightarrow x=0$

$f'(x)$ DNE when bottom $=0$,
but x^2+1 is never zero

(I) Find all critical points of f .

(a) x = - 2 ; (b) x = - 1 ; (c) x = 2 ; (d) x = 0 ;

(e) x = 1 ; (f) NONE OF THE PREVIOUS ANSWERS.

(II) On what interval (s) is f increasing?

(a) $(-2, +\infty)$; (b) $(-1, +\infty)$; (c) $(-1, 1)$; (d) $(-1, 0)$;

(e) $(-\infty, 0)$; (f) $(0, +\infty)$ (g) NONE OF THE PREVIOUS ANSWERS.

		0	
x	$-$		$+$
$1+x^2$	$+$		$+$
$f'(x)=\frac{x}{x^2+1}$	$-$		$+$

$\rightarrow f'(x)>0$ for x in $(0,\infty)$

4. (CONTINUED)

(III) $\boxed{\text{The function f has a local maximum at}}$

(a) x = -2 ; (b) x = -1 ; (c) x = 0 ; (d) x = 1 ;

(e) x = 2 ; $\boxed{\text{(f) The function f has } \underline{\text{no}} \text{ local maxima.}}$

[f' changes from − to +, so there is a local min at $x=0$ but no local max's]

(IV) $\boxed{\frac{d}{dx} f(g(x)) \text{ at } x = 1.}$

$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$. At $x=1$, $f'(g(1)) \cdot g'(1) = f'(2) \cdot 4 = \frac{2}{5} \cdot 4$

(a) 2 ; (b) $\frac{2}{5}$; $\boxed{\text{(c) } \frac{8}{5} \text{ ;}}$ (d) 1 ;

(e) 0 ; (f) NONE OF THE PREVIOUS ANSWERS.

(V) $\boxed{g^{-1}(2)} = 1$ because $g(1) = 2$ [And g is one-to-one]

(a) 3 ; $\boxed{\text{(b) 1 ;}}$ (c) $\frac{1}{3}$; (d) 4;

(e) DOES NOT EXIST ; (f) NONE OF THE PREVIOUS ANSWERS.

(VI) $\boxed{\frac{d}{dx} g^{-1}(x) \text{ at } x = 2.}$ $\frac{d}{dx} g^{-1}(x) = \frac{1}{g'(g^{-1}(x))}$

(a) 1 ; (b) 4 ; $\boxed{\text{(c) } \frac{1}{4} \text{ ;}}$ (d) $\frac{1}{5}$;

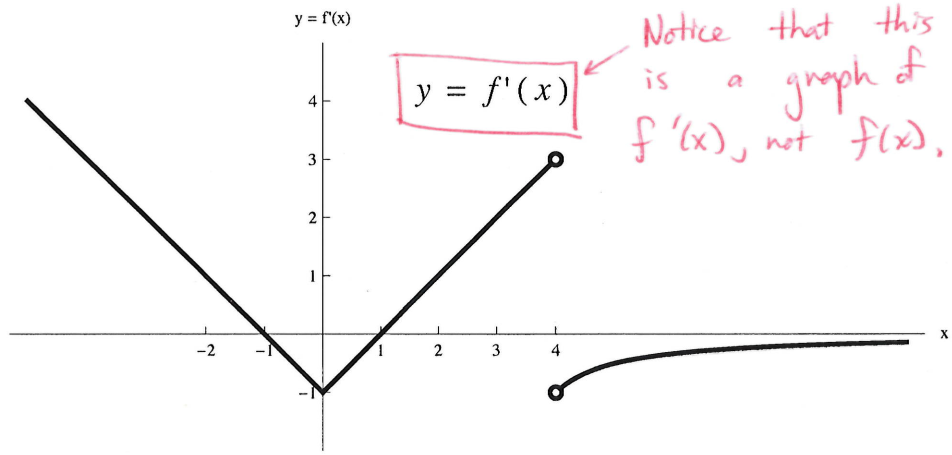
(e) 0 ; (f) NONE OF THE PREVIOUS ANSWERS.

$\frac{1}{g'(g^{-1}(2))} = \frac{1}{g'(1)} = \frac{1}{4}$

5. (18 pts) **EXPLANATION IS NOT REQUIRED, AND NO PARTIAL CREDIT WILL BE GIVEN.**

The graph of **f '** (derivative of **f**) is shown in the figure.

Assume that the function **f** is continuous on $(-\infty, +\infty)$.



Notice that this is a graph of $f'(x)$, not $f(x)$.

Use the given graph of f ' to answer the following questions about f :

(a) (4 pts) On what interval (or intervals) is f increasing?

$(-\infty, -1), (1, 4)$ [these are the intervals where $f'(x) > 0$]

(b) (4 pts) Find the critical points of f.

$x = -1, 1, 4$ [points where $f'(x) = 0$ or $f'(x) = \text{DNE}$ but where $f(x)$ makes sense]

(c) (4 pts) Which critical point (or points) correspond to local maxima?

$x = -1, x = 4$ [points where f' changes from positive to negative]

(d) (3 pts) On what interval (or intervals) is f concave down?

$(-\infty, 0)$ [f concave down $\Leftrightarrow f'' > 0 \Leftrightarrow f'$ is decreasing]

(e) (3 pts) At what point (or points) does f have an inflection point?

$x = 0$ [point where f changes concavity $\Updownarrow$ point where f' changes from increasing to decreasing or decreasing to increasing]