Math 1151
MIDTERM 3
April 9, 2014
Form C

NAME : KEY

OSU Name.# :

Lecturer::

Recitation Instructor :

Recitation Time :

INSTRUCTIONS

- SHOW ALL WORK in problems 1, 2, and 5.
  Incorrect answers with work shown may receive partial credit, but unsubstantiated correct answers may receive NO credit.

  You don't have to show work in problems 3 and 4.

- Give EXACT answers unless asked to do otherwise.

- You do not need to simplify numerical answers such as $\frac{5}{\sqrt{8}} - \frac{3}{\sqrt{32}}$.

- Calculators are permitted EXCEPT those calculators that have computer algebra systems (CAS) or ability to communicate with others. Furthermore, all memory must be cleared and all apps must be removed. PDA's, laptops, and cell phones are prohibited. Do not have these devices out!

- The exam duration is 55 minutes.

- The exam consists of 5 problems starting on page 2 and ending on page 8. Make sure your exam is not missing any pages before you start.

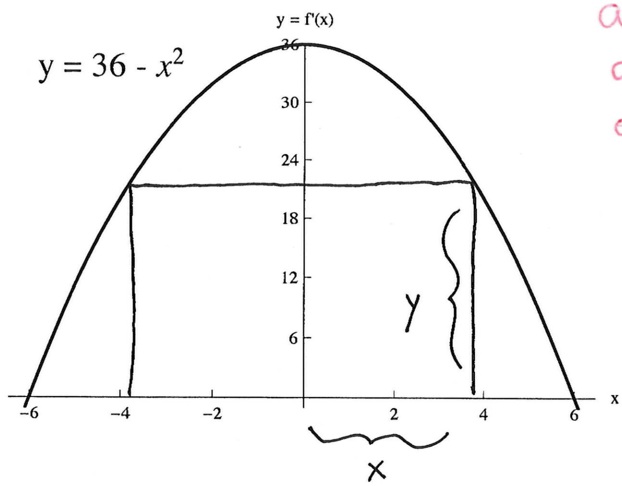
PROBLEM NUMBER	SCORE
1	(20)
2	(20)
3	(22)
4	(18)
5	(20)
TOTAL	(100)

MIDTERM 3
Form C, Page 2

1. (20 pts)

A rectangle is constructed with its base on the x - axis and two of its vertices on the parabola $y = 36 - x^2$ and above the x - axis.

(I) Make a sketch and label it.



Remember that the absolute extrema of a continuous function over a closed interval [a,b] occurs at either a critical point or an endpoint.

(II) What are the dimensions of the rectangle with the maximum area?

Justify your answer!

$\text{Area} = 2x \cdot y = 2x(36 - x^2) = 72x - 2x^3$

we want to maximize $A(x) = 72x - 2x^3$ for x in $[0,6]$.

$A'(x) = 72 - 6x^2$

$A' = 0 \Rightarrow 72 - 6x^2 = 0$

$72 = 6x^2$

$12 = x^2$

$\pm\sqrt{12} = x$

$x = \sqrt{12}$ is only critical point in $(0,6)$.

$A(0) = 0$

$A(\sqrt{12}) = 72\sqrt{12} - 2(\sqrt{12})^3 = 72\sqrt{12} - 24\sqrt{12} = 48\sqrt{12} > 0$

$A(6) = 0$.

$\Rightarrow$ Global max occurs when $x = \sqrt{12}$ and $y = 36 - (\sqrt{12})^2 = 24$

(III) What is that maximum area?

Maximum area $= A(\sqrt{12}) = 48\sqrt{12}$.

You could also justify that an absolute max occurs at $x = \sqrt{12}$ using either the 1st or 2nd derivative test since there is only one critical point in $[0,6]$.

This tells us what to do first. Namely, this step

provided this limit exists.

2. (20 pts)

indeterminate form

$\downarrow$

$(1+0)^{\infty} = 1^{\infty}$

(I) (10 pts) Evaluate the limit . You may use L ' Hospital ' s Rule.

$$\lim_{x \to 0^+} (1 + 3x)^{\frac{4}{x}} = e^{L}, \text{ where } L = \lim_{x \to 0^+} \frac{4}{x} \cdot \ln(1+3x)$$

$\frac{4 \ln 1}{0} = \frac{0}{0}$ ← good to use L'hôpital

$$L = \lim_{x \to 0^+} \frac{4\ln(1+3x)}{x} \underset{\text{L'hôp}}{=} \lim_{x \to 0^+} \frac{4 \cdot \frac{1}{1+3x} \cdot 3}{1} = \frac{4 \cdot \frac{1}{1} \cdot 3}{1} = 12.$$

$$\Rightarrow \lim_{x \to 0^+} (1+3x)^{\frac{4}{x}} = e^{12}$$

(II) (10 pts) Given the acceleration function of an object moving along a line, find the <u>position</u> <u>function</u> with the given initial velocity and position.

a (t) = 4 t, v (0) = 3, s (0) = 5

$v(t) = \int a(t)\,dt = \int 4t\,dt = 2t^2 + C$

$3 = v(0) = 2 \cdot 0^2 + C \to C = 3$

$v(t) = 2t^2 + 3$

$s(t) = \int v(t)\,dt = \int 2t^2 + 3\,dt = \frac{2}{3}t^3 + 3t + C$

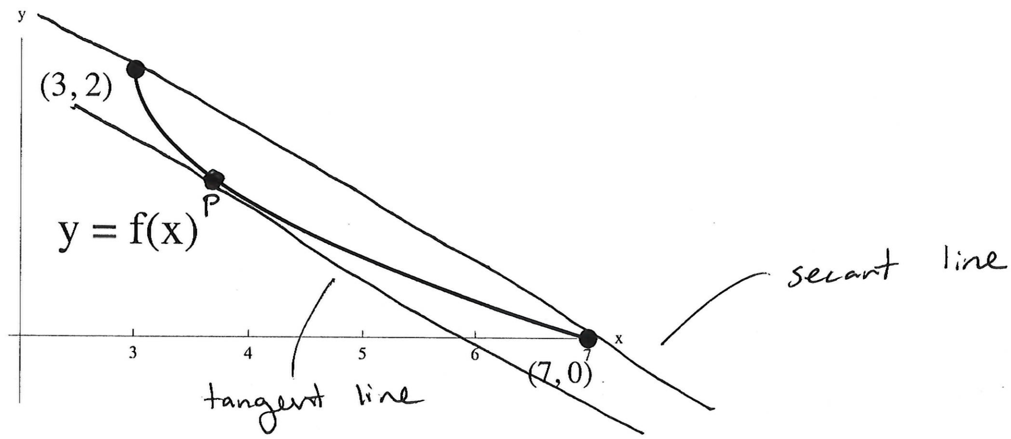
$5 = s(0) = \frac{2}{3} \cdot 0^3 + 3 \cdot 0 + C \to C = 5$

$$\boxed{s(t) = \frac{2}{3}t^3 + 3t + 5}$$

3. (22 pts)

(I) (10 pts) **EXPLANATION IS NOT REQUIRED, AND NO PARTIAL CREDIT WILL BE GIVEN.**

The figure shows the graph of a function f on the interval [3, 7].



(a) Find the average rate of change of the function f on the interval [3, 7].

Average rate of change $= \dfrac{f(7) - f(3)}{7 - 3} = \dfrac{0-2}{4} = -\dfrac{1}{2}$

(b) Find the slope of the secant line that passes through (3, 2) and (7, 0).

Slope $= \dfrac{0-2}{7-3} = -\dfrac{1}{2}$ (same as average rate of change of f on [3,7])

(c) Sketch the secant line from the part (b) in the figure above.

(d) In the figure above mark the points P (if they exist) at which the slope of the tangent line equals the slope of the secant line from the part (b).

→ [tangent line parallel to secant line]

(e) Name the theorem that guarantees that such a point P exists.

Mean Value Theorem

(f) Sketch the tangent line at P in the figure above.

3. (CONTINUED) MULTIPLE CHOICE! CIRCLE THE CORRECT ANSWER IN EACH PART.

(II) (3 pts)

$f(0) = e^0 = 1$

$f'(x) = 2e^{2x}$

Find the equation of the line that represents the <u>linear approximation</u> to the function $f(x) = e^{2x}$ at $a = 0$. $L(x) = f(0) + f'(0)(x-0)$

$f'(0) = 2$

(a) $y = 1 + 2x$; (b) $y = 1 - 2x$; (c) $y = \ln(2x + 1)$; (d) $y = x$;

(e) SUCH A LINE DOES NOT EXIST; (f) NONE OF THE PREVIOUS ANSWERS.

(III) (3 pts)

Determine the indefinite integral $\int \left(\frac{1}{1 + x^2} - 3 \right) dx$.

$= \tan^{-1} x - 3x + C$

Remember that $(\tan^{-1} x)' = \frac{1}{1+x^2}$

(a) $\ln(1 + x^2) + C$; (b) $\ln(1 + x^2) - 3x + C$; (c) $\tan^{-1} x - 3x + C$;

(d) $\tan^{-1} x$; (e) $\tan^{-1} x + C$; (f) NONE OF THE PREVIOUS ANSWERS.

(IV) (3 pts)

Evaluate the sum $\sum_{k=1}^{3} (1 + k^2)$.

$= (1+1^2) + (1+2^2) + (1+3^2) = 2 + 5 + 10$

(a) 7; (b) 15; (c) 17; (d) $\frac{k(k+1)(2k+1)}{6}$;

(e) 16; (f) NONE OF THE PREVIOUS ANSWERS.

(V) (3 pts)

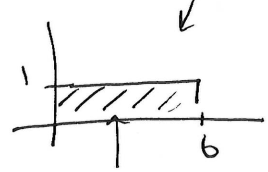
Given that $\int_0^6 g(x)\, dx = 2$, evaluate the integral $\int_0^6 (4g(x) + 1)\, dx$.

$= 4\int_0^6 g(x)dx + \int_0^6 1dx$

$= 4 \cdot 2 + \int_0^6 1dx$

(a) 2; (b) 8; (c) 26; (d) 54; (e) 14;

(f) NONE OF THE PREVIOUS ANSWERS.



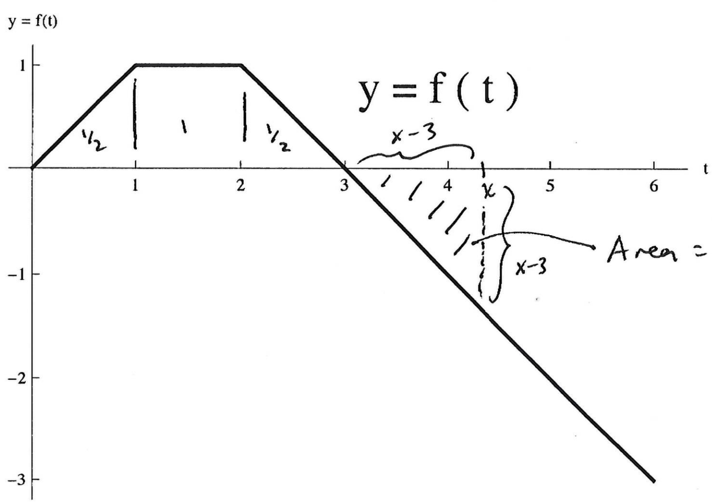
Area $= 1 \cdot 6 = 6$

4. (18 pts) MULTIPLE CHOICE ! CIRCLE THE CORRECT ANSWER IN EACH PART.

The graph of f is shown in the figure. Let

$$A(x) = \int_0^x f(t)\,dt, \text{ for } 0 \le x \le 6, \quad \text{and} \quad F(x) = \int_3^x f(t)\,dt, \text{ for } 3 \le x \le 6.$$

be two area functions for f.



$\rightarrow$ Area $= \frac{(x-3)^2}{2}$ Net Area $= -\frac{(x-3)^2}{2}$

(I) (3 pts) $\boxed{\text{Evaluate } A(3).}$ $= \int_0^3 f(t)\,dt = \frac{1}{2} + 1 + \frac{1}{2}$

(a) -2; $\boxed{\text{(b) } 2;}$ (c) 1; (d) -1; (e) 0;

(f) $-\frac{3}{2}$; (g) $\frac{3}{2}$ (h) NONE OF THE PREVIOUS ANSWERS.

(II) (3 pts) $\boxed{\text{Evaluate } F(3).}$ $= \int_3^3 f(t)\,dt = 0$

(a) 2; (b) 1; $\boxed{\text{(c) } 0;}$ (d) $\frac{3}{2}$;

(e) $\frac{1}{2}$; (f) NONE OF THE PREVIOUS ANSWERS.

Fundamental Theorem of Calculus

4. (18 pts) MULTIPLE CHOICE ! CIRCLE THE CORRECT ANSWER IN EACH PART.

(III) (3 pts) $\boxed{\text{Evaluate } A'(1.5)}$ $= f(1.5) = 1$

(a) 0; (b) - 1 ; $\boxed{\text{(c) } 1;}$ (d) $\frac{3}{2}$;

(e) $\frac{1}{2}$; (f) NONE OF THE PREVIOUS ANSWERS.

(IV) (3 pts) $\boxed{\text{Find an expression for } F'(x) \text{, for } 3 \le x \le 6.}$

$F'(x) = f(x)$

On $3 \le x \le 6$, $y = f(x)$ is a line with slope -1 passing through $(3,0)$.

$y - 0 = -1\cdot(x-3)$

$y = -x+3$

(a) $F'(x) = 1$; $\boxed{\text{(b) } F'(x) = 3 - x;}$ (c) $F'(x) = \frac{-(3-x)^2}{2}$;

(d) $F'(x) = -(3-x)^2$; (e) NONE OF THE PREVIOUS ANSWERS.

(V) (3 pts) $\boxed{\text{Find an expression for } F(x) \text{, for } 3 \le x \le 6.}$

$F(x) = \int_3^x f(t)\,dt$ [look at ~~the~~ graph].

(a) $F(x) = 1$; (b) $F(x) = 3 - x$; $\boxed{\text{(c) } F(x) = \frac{-(3-x)^2}{2}}$;

(d) $F(x) = -(3-x)^2$; (e) NONE OF THE PREVIOUS ANSWERS.

(VI) (3 pts) $\boxed{\text{Evaluate } \int_0^5 |f(t)|\,dt.}$ $= \frac{1}{2} + 1 + \frac{1}{2} + \frac{1}{2}\cdot 2\cdot 2 = 4$

(a) 0; (b) 6 ; $\boxed{\text{(c) } 4;}$ (d) 2 ;

(e) 1 ; (f) NONE OF THE PREVIOUS ANSWERS.

remember that $\int_a^b f(x)dx$ is the <u>net area</u> between the graph of $y = f(x)$ and x-axis while $\int_a^b |f(x)|dx$ is the <u>area</u> between $y = f(x)$ and x-axis.

5. (20 pts)

(a) Using the Fundamental theorem of calculus, evaluate the integral $\int_0^6 (4x - x^2)\, dx$.

$$= 2x^2 - \frac{1}{3}x^3 \Big|_0^6$$

$$= 2 \cdot 6^2 - \frac{1}{3} \cdot 6^3 - (0 - 0)$$

$$= 2 \cdot 36 - 2 \cdot 36 = 0$$

(b) Use the midpoint Riemann sum with n = 3 to approximate the value of the integral $\int_0^6 (4x - x^2)\, dx$.

$$\Delta x = \frac{6-0}{3} = 2$$

Midpoint Riemann Sum $= f(1) \cdot \Delta x + f(3) \cdot \Delta x + f(5) \cdot \Delta x$

$$= (4 \cdot 1 - 1^2) \cdot 2 + (4 \cdot 3 - 3^2) \cdot 2 + (4 \cdot 5 - 5^2) \cdot 2$$

$$= 3 \cdot 2 + 3 \cdot 2 - 5 \cdot 2 = 2$$

(c) Illustrate the midpoint Riemann sum from the part (b) by sketching the appropriate rectangles in the figure below.