

## L'HÔPITAL'S RULE

1) What does 'Indeterminate Form' mean? What are the basic types?

2) Evaluate the limits.

$$\lim_{x \rightarrow 2} \frac{x-2}{x^2-4}, \quad \lim_{x \rightarrow 1} \frac{\ln x}{x-1}$$

3) What formula do we have to help evaluate Indeterminate Forms? Which types can it be used for directly? How do we deal with the other forms?

4) Evaluate the limits.

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{x^3 + 3x^2}{e^x - x - 1}, & \quad \lim_{x \rightarrow \infty} \frac{\ln(3x+1)}{1 + \ln(5x+2)}, \\ \lim_{x \rightarrow 0} \frac{e^x + 2}{3x + 1}, & \quad \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{1 + \tan x}{\sec x} \end{aligned}$$

5) How do we deal with  $f(x) \cdot g(x)$  if it gives a  $0 \cdot \infty$  form? Evaluate the following limits.

$$\lim_{x \rightarrow 0} x^2 \csc x, \quad \lim_{x \rightarrow \infty} x \tan\left(\frac{1}{x}\right), \quad \lim_{x \rightarrow -\infty} x^2 e^x$$

6) How to deal with  $f(x) - g(x)$  if it gives an  $\infty - \infty$  form? Evaluate the following limits.

$$\lim_{x \rightarrow 0^+} (\csc x - \cot x), \quad \lim_{x \rightarrow \infty} (x - \sqrt{x^2 - x})$$

7) How to deal with  $f(x)^{g(x)}$  when it gives either  $1^\infty$ ,  $0^0$ , or  $\infty^0$  form? Evaluate the following limits.

$$\lim_{x \rightarrow 0^+} x^x, \quad \lim_{x \rightarrow \infty} (e^x + x)^{\frac{1}{x}}, \quad \lim_{x \rightarrow \infty} \left(1 + \frac{a}{x}\right)^x$$

8) A particular zombie colony is found to have population size  $P(t) = 2.5t^2e^{-0.3t}$ . In the long run, will they win or will they be defeated?

9) What does it mean that a function  $f$  grows faster than a function  $g$ ? What does it mean that they have comparable growth?

10) Does  $(\ln x)^2$  or  $x^{1/2}$  grow faster?  $3x^5 - 2x^2 + 7$  vs.  $x^5$ ?  $4^x$  vs  $x^x$ ?

11) For any positive numbers  $p, q, b$  with  $b > 1$ , we have growth rates

$$(\ln x)^q \ll x^p \ll b^x \ll x^x$$

Which grows faster,  $x^2 \ln^3 x$  or  $x^{2.1}$ ?