SAMPLE MIDTERM 1 - PROBLEMS

READ THIS NOTE: I will be using parenthesis "(", ")" and brackets "[", "]" interchangeably (when there are too many parenthesis involved, I will put brackets to clear the situation a bit out, so you can see where one begins and where one ends an expression).

Also, I will be using exclusively the notation y', $f'(x)$, $h'(z)$ etc for the derivative. This doesn't, certainly, mean that notations such as $\frac{dy}{dx}$, $\frac{df}{dx}$ etc are not used, or invalid. If you prefer using the latter notation, kindly replace, without any penalty, accordingly: y' with $\frac{dy}{dx}$, $f'(x)$ with $\frac{df}{dx}$, etc.

Any comments or corrections regarding these solutions should be immediatly directed to me:

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Good luck!

(1) Compute the following limits. If the limit is $+\infty$, $-\infty$ or doesn't exist, then say so. Leave answers in fractions.

(a) $\lim\limits_{x \to 4^+} \dfrac{x^2 - 4}{x + 4}$

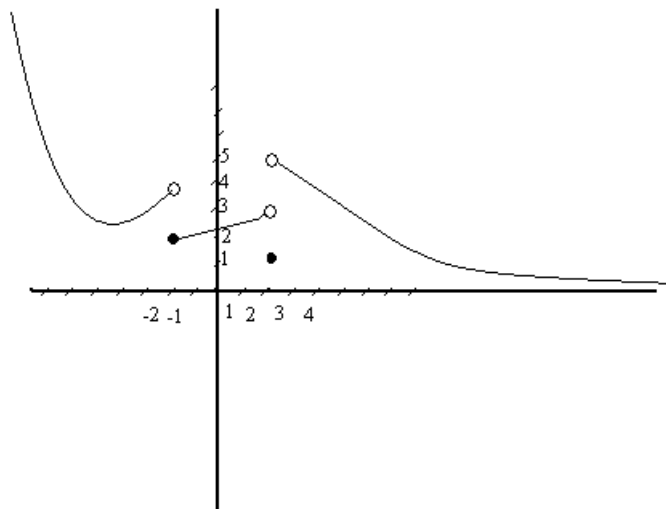(b) $\lim\limits_{x \to 2^+} \dfrac{x - 9}{x^2 - 4}$

(c) $\lim\limits_{x \to -\infty} \dfrac{8x^2 - 4x - 6x^3}{5x + 2x^2}$

(d) $\lim\limits_{x \to -2} \dfrac{x^2 - x - 6}{x^2 - 4}$

(e) $\lim\limits_{x \to \infty} \dfrac{12x^3 - 4x - 3}{7 - 5x - 4x^3}$

(f) use the definition of the derivative ONLY, to find $f'(x)$, for $f(x) = \dfrac{1}{5x + 3}$ (find $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$).

(g) given the graph of f(x) find the limits:

(i) $\lim\limits_{x \to 2^+} f(x)$

(ii) $\lim\limits_{x \to 2^-} f(x)$

(iii) $f(2)$

(iv) $\lim\limits_{x \to \infty} f(x)$

(v) $\lim\limits_{x \to -\infty} f(x)$

(2) Let

$$f(x) = \begin{cases} \frac{12}{x+2} & \text{if } x < 1 \\ 2 & \text{if } x = 1 \\ \frac{12}{x+3} & \text{if } x > 1 \end{cases}$$

(a) find $\lim\limits_{x \to 1^+} f(x)$

(b) find $\lim\limits_{x \to 1^-} f(x)$

(c) find $\lim\limits_{x \to -2^+} f(x)$

(d) find $\lim\limits_{x \to -3^-} f(x)$

(3) Find the derivatives of the following functions (do not simplify)

   (a) $f(x) = (7x + 8x^6)$

   (b) $f(t) = \dfrac{7}{4t^3}$

   (c) $y = (x^5 - 7x + 3)x^{8/5}$

(4) Solve the inequality
$$\frac{(x-9)(2-x)}{x+5} \le 0$$

(5) The path (graph) of a projectile is given by $y = x + \frac{1}{4}x^2$. Find an equation of the tangent line to this curve at the point $(2, 3)$.

(6) If a manufacturer's cost function is given by $C = .3q^2 + 2q + 850$
   (a) find the marginal cost function. Also
   (b) find the marginal cost when 3 units are produced?