SAMPLE - MIDTERM 3

READ THIS NOTE: I will be using parenthesis "(", ")" and brackets "[", "]" interchangeably (when there are too many parenthesis involved, I will put brackets to clear the situation a bit out, so you can see where one begins and where one ends an expression).

Also, I will be using exclusively the notation y', $f'(x)$, $h'(z)$ etc for the derivative. This doesn't, certainly, mean that notations such as $\frac{dy}{dx}$, $\frac{df}{dx}$ etc are not used, or invalid. If you prefer using the latter notation, kindly replace, without any penalty, accordingly: y' with $\frac{dy}{dx}$, $f'(x)$ with $\frac{df}{dx}$, etc.

Any comments or corrections regarding these solutions should be immediatly directed to me:

cosmin@math.ohio-state.edu

Good luck!

(1) For the function $f(x) = x + \frac{4}{x}$
   (a) find all critical points for $f(x)$
   (b) use derivatives to determine the interval(s) (if any) on which $f(x)$ is increasing and the interval(s) (if any) on which $f(x)$ is decreasing (if there are none, please say so)
   (c) use information obtained in part (a) to find the values of x for which $f(x)$ has relative max and relative min (if there are none please say so)

(2) Let $f(x) = x^4 - 4x^2 + 9$.
   (a) find its y-intercept
   (b) use derivatives to find the interval(s) where $f(x)$ is increasing and where $f(x)$ is decreasing
   (c) use information obtained in part (b) to find its points of relative max and relative min
   (d) use derivatives to determine the interval(s) where it is concave up and where it is concave down
   (e) where are its point(s) of inflection?
   (f) sketch a graph of the function $f(x)$ showing all the information obtained in parts (a)-(e); labeling the points of relative extrema and also the inflection points

Date: March 8, 2003.

(3) For the graph of the function

$$f(x) = \frac{2x^2 - 8}{x + 9}$$

(a) find the x-intercept(s) and the y-intercept if there are any. If there are none please say so.

(b) find all its horizontal asymptote(s). If there are none please say so (Show all work)

(c) find all its vertical asymptote(s). If there are none please say so (show all work)

(d) sketch a graph of the function $f(x)$ using information obtained in parts (a) and (b) above

(4) (a) use the second derivative test to find point(s) of relative max and relative min for the function

$$f(x) = 2x^3 + 5x^2 - 4x + 15$$

(b) find the absolute max and absolute min that occur for the function in part (a) ($f(x) = 2x^3 + 5x^2 - 4x + 15$) in the interval [0.3].

(5) (a) use derivatives only to determine the interval(s) where the graph of

$$f(x) = x^4 - 4x^3 + 15$$

is concave up and where it is concave down.

(b) use the information obtained in part (a) to find its points of inflection