SAMPLE FINAL EXAM - PROBLEMS

READ THIS NOTE: I will be using parenthesis "(", ")" and brackets "[", "]" interchangeably (when there are too many parenthesis involved, I will put brackets to clear the situation a bit out, so you can see where one begins and where one ends an expression).

Also, I will be using exclusively the notation y', $f'(x)$, $h'(z)$ etc for the derivative. This doesn't, certainly, mean that notations such as $\frac{dy}{dx}$, $\frac{df}{dx}$ etc are not used, or invalid. If you prefer using the latter notation, kindly replace, without any penalty, accordingly: y' with $\frac{dy}{dx}$, $f'(x)$ with $\frac{df}{dx}$, etc.

Any comments or corrections regarding these solutions should be immediatly directed to me:

cosmin@math.ohio-state.edu

Good luck!

(1) Compute the following limits. If the limit is $+\infty$ or $-\infty$ or does not exist then say so.

(a) $\displaystyle \lim_{x\to 0} \frac{(x+3)^2-9}{x}$

(b) $\displaystyle \lim_{x\to 4^+} \frac{4x}{16-x^2}$

(c) $\displaystyle \lim_{t\to 4} \frac{t^2-2t-8}{9t}$

(d) $\displaystyle \lim_{h\to 0} \frac{\frac{4}{5+h}-\frac{4}{5}}{h}$

(e) $\displaystyle \lim_{x\to\infty} \frac{6x^6-7x+9}{9-8x^6}$

(f) $\displaystyle \lim_{t\to-\infty} \frac{9+7e^t}{8-9t}$

(g) $\displaystyle \lim_{x\to 1} \frac{x^2+4x-5}{2x^2+x-3}$

(2) Let $f(x) = \begin{pmatrix} 3-2x & \text{if} & x>1 \\ x^2 & \text{if} & 0\le x\le 1 \\ \frac{4x}{x-9} & \text{if} & x<0 \end{pmatrix}.$

(a) Find $\displaystyle \lim_{x\to 1^+} f(x) =$

(b) Find $\displaystyle \lim_{x\to 1^-} f(x) =$

(c) Find all values of x for which $f(x)$ is not continuous.

Date: March 15, 2003.

(3) Use implicit differentiation to express $\frac{dy}{dx}$ in terms of x and y:

$$5x^4 - 6x^5y + 8y^7 = 17$$

(4) Find the derivatives of the following functions. Please don't simplify.

(a) $f(u) = (7u^5 - 1)^8(u^2 + 9u + 1)$

(b) $y = \dfrac{x+7}{x^5 - 8}$

(c) $y = x^5 \cdot e^{-9x}$

(d) $y = \ln[(7x+6)^6(4x-3)^9e^{8x}]$

(e) $y = (5x)^{9/5} + 5(x)^{9/5}$

(f) $f(x) = \ln(4x^9) + [\ln(4x)]^9$

(g) $y = \ln[x^{(7x-5)}]$

(5) Solve the inequality:

$$\frac{(5-x)(4+x)}{9-x} \leq 0$$

(6) If $y = e^{6x+4}$ then find $\frac{d^2y}{dx^2}$.

(7) Use logarithmic differentiation to find $\frac{dy}{dx}$ where

$$y = \frac{(8x^5 - 9x + 5)^5 \cdot (x^4 - 2x + 1)^7}{(x^8 - 6x + 1)}.$$

(8) (a) Find the slope of the tangent to the graph of $y = x^6 - 5x + 4$ at the point $(1, 0)$.
(b) Find the equation of the tangent line to the graph in part (a) above.

(9) Use derivatives including 2^{nd} derivative test (Cosmin's note: ???) and end points to find the points of absolute extrema for the function $f(x) = 2x^3 - 3x^2 - 36x + 8$ in the interval $[-5, 5]$.

(10) Determine the concavity and the x-values where the points of inflection occur for

$$y = -\frac{x^4}{4} + \frac{9x^2}{2} + 2x$$

(11) Let $y = f(x) = \dfrac{x^2-4}{x^2-1}$. Given $y' = \dfrac{6x}{(x^2-1)^2}$ and $y'' = \dfrac{-(18x^2+6)}{(x^2-1)^3}$

(a) Find its y-intercept
(b) Find its x-intercept(s)
(c) Find its horizontal asymptotes
(d) Find its vertical asymptotes
(e) Using derivatives only, determine the interval(s) where the graph of $y = f(x)$ is increasing and where it is decreasing
(f) Use information in part (e) to find points of relative maxima and relative minima (if there are none, please say so)
(g) Using derivatives only, determine the interval(s) where the graph of $y = f(x)$ is concave up and where it is concave down.
(h) What is (are) its point(s) of inflection?
(i) Sketch a graph of the function $y = f(x)$ showing all the information obtained in parts (a)-(e), labeling the intercepts, points of relative extrema and of inflection.

(12) **Cost** A manufacturer has determined that, for a certain product, the average cost (in dollars per unit) is given by

$$\overline{c} = 2q^2 - 36q + 210 - \frac{200}{q}$$

where $2 \leq q \leq 10$. At what level within the interval $[2, 10]$ should production be fixed in order to minimize total cost?

(13) Find two non negative numbers whose sum is 60 and such that the product of the two numbers will be a maximum.