SAMPLE MIDTERM 1 - SOLUTIONS

READ THIS NOTE: I will be using parenthesis "(", ")" and brackets "[", "]" interchangeably (when there are too many parenthesis involved, I will put brackets to clear the situation a bit out, so you can see where one begins and where one ends an expression).

Also, I will be using exclusively the notation y', $f'(x)$, $h'(z)$ etc for the derivative. This doesn't, certainly, mean that notations such as $\frac{dy}{dx}$, $\frac{df}{dx}$ etc are not used, or invalid. If you prefer using the latter notation, kindly replace, without any penalty, accordingly: y' with $\frac{dy}{dx}$, $f'(x)$ with $\frac{df}{dx}$, etc.

Any comments or corrections regarding these solutions should be immediatly directed to me:

cosmin@math.ohio-state.edu

Good luck!

(1) (a) plug in 4 - works!

$$\lim_{x\to 4^+} \frac{x^2-4}{x+4} = \frac{12}{8}$$

(b) plug in 2 - get -7/0; it is an infinity (vertical asymptote ...), so must check the sign, to the right of 2; choose 2.5: $-6.5/2.25$ =negative, so

$$\lim_{x\to 2^+} \frac{x-9}{x^2-4} = -\infty$$

(c) limit to infinity; scan the numerator and denominator for highest powers, and drop everything else:

$$\lim_{x\to-\infty} \frac{8x^2-4x-6x^3}{5x+2x^2} = \lim_{x\to-\infty} \frac{-6x^3}{2x^2} =$$
$$= \lim_{x\to-\infty} -3x = \infty$$

(d) plug in -2; obtain 0/0; must cancel the common 0; the factor giving the zero is $x-(-2) = x+2$

$$\lim_{x\to-2} \frac{x^2-x-6}{x^2-4} = \lim_{x\to-2} \frac{(x+2)(x-3)}{(x-2)(x+2)} =$$
$$= \lim_{x\to-2} \frac{x-3}{x-2} = \frac{-5}{-4} = \frac{5}{4}$$

Date: March 16, 2003.

(e) again, drop the lower powers:

$$\lim_{x\to\infty} \frac{12x^3 - 4x - 3}{7 - 5x - 4x^3} = \lim_{x\to\infty} \frac{12x^3}{-4x^3} = \\ = \lim_{x\to\infty} -3 = -3$$

(f) we have:

$$f(x+h) = \frac{1}{5(x+h)+3}$$

$$f(x+h) - f(x) = \frac{1}{5(x+h)+3} - \frac{1}{5x+3} =$$

$$= \frac{5x+3}{(5x+3)[5(x+h)+3]} - \frac{5(x+h)+3}{(5x+3)[5(x+h)+3]} =$$

$$= \frac{5x+3-(5x+5h+3)}{(5x+3)[5(x+h)+3]} = \frac{5x+3-5x-5h-3)}{(5x+3)[5(x+h)+3]} =$$

$$= \frac{-5h}{(5x+3)[5(x+h)+3]}$$

so, we eventually get:

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{-5h}{(5x+3)[5(x+h)+3]} \cdot \frac{1}{h} = \\ = \lim_{h\to 0} \frac{-5}{(5x+3)[5(x+h)+3]} = \frac{-5}{(5x+3)^2}$$

(g) for limits always ignore the VALUE in the point given and also the part of the function which is not used; we obtain:

(i) $\lim_{x\to 2^+} f(x) = 5$

(ii) $\lim_{x\to 2^-} f(x) = 3$

(iii) $f(2) = 1$

(iv) $\lim_{x\to\infty} f(x) = 0$

(v) $\lim_{x\to-\infty} f(x) = \infty$

(2) same as above; here we ignore the value and the FORMULA that is not used (look at what x-s are involved)

(a) $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \frac{12}{x+3} = 12/4 = 3$

(b) find $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \frac{12}{x+2} = 12/3 = 4$

(c) find $\lim_{x\to -2^+} f(x) = \lim_{x\to -2^+} \frac{12}{x+2} = \infty$
(why? plugging in -2 - which by the way is LESS THAN 1, and that's why we use that formula - we get 12/0, which points to an infinity; since we are to the RIGHT of -2, plugging in -1.5 gives us a positive amount, 12/.5, and so the infinity is positive)

(d) find $\lim_{x\to -3^-} f(x) = \lim_{x\to -3^-} \frac{12}{x+2} = 12/(-1) = -12$

(3) (a) $f'(x) = (7x + 8x^6)' = 7 + 8 \cdot 6x^5 = 7 + 48x^5$

(b) $f'(t) = (\frac{7}{4} \cdot t^{-3})' = \frac{7}{4} \cdot (-3)t^{-4}$

(c)

$$y' = [(x^5 - 7x + 3)x^{8/5}]' = (x^{5+8/5} - 7x^{1+8/5} + 3x^{8/5})' =$$

$$= (5 + 8/5)x^{5+8/5-1} - 7 \cdot (1 + 8/5)x^{1+8/5-1} + 3 \cdot 8/5x^{8/5-1}$$

(4) Draw a table, point out where the numerator and denominator become zero, and check sign between these values:

x		-5		2		9	
sign	+	DNE	$-$	0	+	0	$-$

Need less than OR EQUAL to zero - which means we also involve the endpoints which give 0 (NOT those that give DNE!). So, the answer is: $(-5, 2]$ and $[9, \infty)$.

(5) The slope is given by the derivative: $y' = \frac{1}{4} \cdot 2x = x/2$; plug in the x-value, which is 2:

$$m = 2/2 = 1$$

Now we have the slope, $m = 1$ and the point, $(2, 3)$; ignore what we started with, concentrate only on the equation of the line:

$$y - 3 = 1 \cdot (x - 2)$$

(6) (a) marginal cost function is the derivative of the cost: $c' = .3 \cdot 2q + 2 = .6q + 2$

(b) $q = 3$; plug it into the above formula: $c'(3) = .6 \cdot 3 + 2 = 1.8 + 2 = 3.8$