REVIEW PROBLEMS - SOLUTIONS

READ THIS NOTE: I will be using parenthesis "(", ")" and brackets "[", "]" interchangeably (when there are too many parenthesis involved, I will put brackets to clear the situation a bit out, so you can see where one begins and where one ends an expression).

Also, I will be using exclusively the notation y', $f'(x)$, $h'(z)$ etc for the derivative. This doesn't, certainly, mean that notations such as $\frac{dy}{dx}$, $\frac{df}{dx}$ etc are not used, or invalid. If you prefer using the latter notation, kindly replace, without any penalty, accordingly: y' with $\frac{dy}{dx}$, $f'(x)$ with $\frac{df}{dx}$, etc.

Any comments or corrections regarding these solutions should be immediatly directed to me:

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Good luck!

(1) Find the derivatives of the following functions (Do not simplify)

(a)
$$f(x) = (2 + 15x - 7x^2)(x^9 - 8x + 4)$$

solution:
$$f'(x) = (15 - 7 \cdot 2x) \cdot (x^9 - 8x + 4) + (2 + 15x - 7x^2) \cdot (9x^8 - 8)$$

(b)
$$f(t) = \frac{t^3 + 7t + 5}{t^5 - 9t^2 + 3}$$

solution:
$$f'(t) = \frac{(3t^2 + 7) \cdot (t^5 - 9t^2 + 3) - (t^3 + 7t + 5) \cdot (5t^4 - 9 \cdot 2t)}{(t^5 - 9t^2 + 3)^2}$$

(c)
$$y = (8x + 5)^{\frac{4}{7}} + e^{\frac{4}{7}}$$

solution:
$$y' = \frac{4}{7}(8x + 5)^{\frac{4}{7} - 1} \cdot 8$$

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(d)

$$f(x) = \ln[(7x+1)^6] + [\ln(7x+1)]^6$$

solution:

$$f'(x) = \frac{1}{(7x+1)^6} \cdot 6(7x+1)^5 \cdot 7 + 6[\ln(7x+1)]^5 \cdot \frac{1}{7x+1} \cdot 7$$

(e)

$$y = 3^{5t} - \log_3(5t)$$

solution:

$$y' = (3^{5t} \cdot \ln(3)) \cdot 5 - [\frac{1}{5t} \cdot \frac{1}{\ln(3)}] \cdot 5$$

(f)

$$y = e^{9x^5} - 2\sqrt{x}$$

solution:

$$y' = e^{9x^5} \cdot (9 \cdot 5x^4) - 2 \cdot \frac{1}{2} x^{\frac{1}{2}-1}$$

(g)

$$f(x) = x^{7\ln(5)}$$

solution:

$$f'(x) = 7\ln(5)x^{7\ln(5)-1}$$

(h)

$$f(x) = \frac{4 + (2x-3)^{11}}{4 - (2x-3)^{11}}$$

solution:

$$f'(x) = \frac{(11(2x-3)^{10} \cdot 2) \cdot (4 - (2x-3)^{11}) - (4 + (2x-3)^{11}) \cdot (-11(2x-3)^{10} \cdot 2)}{[(4-(2x-3))^{11}]^2}$$

(i)

$$f(x) = \ln[(3x^9+1)^7 \cdot (5x-1)^8]$$

solution: the hard way

$$f'(x) = \frac{1}{(3x^9+1)^7 \cdot (5x-1)^8} \cdot [(7(3x^9+1)^6 \cdot 3 \cdot 9x^8) \cdot (5x-1)^8 + (3x^9+1)^7 \cdot (8(5x-1)^7 \cdot 5)]$$

the easy way, using algebra and properties of logs:

$$f(x) = 7\ln(3x^9+1) + 8\ln(5x-1)$$

$$f'(x) = 7 \cdot \frac{1}{3x^9+1} \cdot (3 \cdot 9x^8) + 8 \cdot \frac{1}{5x-1} \cdot 5$$

(j)

$$y = \sqrt[7]{\frac{9}{x^6-6x+15}}$$

solution: rewrite, using properties of powers

$$y = 9^{\frac{1}{7}} \cdot (x^6-6x+15)^{-\frac{1}{7}}$$

$$y' = 9^{\frac{1}{7}} \cdot (-\frac{1}{7})(x^6-6x+15)^{-\frac{1}{7}-1} \cdot (6x^5-6)$$

(2) Use implicit differentiation to express $\frac{dy}{dx}$ in terms of x and y from

$$9x^6 + 8x^3y^2 - y^9 = 95.$$

solution: rewrite a bit

$$9x^6 + (8x^3) \cdot y^2 - y^9 = 95$$

differentiate both sides

$$9 \cdot 6x^5 + (8 \cdot 3x^2 \cdot y^2 + 8x^3 \cdot 2y \cdot y') - 9y^8 \cdot y' = 0$$

$$\begin{gathered} 54x^5 + 24x^2y^2 + 16x^3y \cdot y' - 9y^8 \cdot y' = 0 \\ 16x^3y \cdot y' - 9y^8 \cdot y' = -54x^5 - 24x^2y^2 \\ (16x^3y - 9y^8) \cdot y' = -54x^5 - 24x^2y^2 \end{gathered}$$

$$y' = \frac{-54x^5 - 24x^2y^2}{16x^3y - 9y^8}$$

(3) Use Chain Rule to find $\frac{dy}{dx}$ where

$$y = u^8 + 3u^6 - 4$$

and

$$u = x^5 - 5x + 16.$$

solution: we have $x \to x^5 - 5x + 16 = u \to u^8 + 3u^6 - 4 = y$ and so we have:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$\frac{dy}{dx} = (8u^7 + 3 \cdot 6u^5) \cdot (5x^4 - 5)$$

Replace u by its formula in terms of x:

$$\frac{dy}{dx} = [8(x^5 - 5x + 16)^7 + 3 \cdot 6(x^5 - 5x + 16)^5] \cdot (5x^4 - 5)$$

(4) Find the equation of the tangent line to the graph of

$$y = x^3 - 7x + 6$$

at the point $(1, 0)$.

solution: slope of tangent line is the derivative

$$y' = 3x^2 - 7$$
$$x = 1 \Rightarrow m = y'(1) = 3 \cdot 1^2 - 7 = 3 - 7 = -4$$

equation of tangent line:

$$y - 0 = (-4)(x - 1)$$

(5) Find the rate of change of $f(x) = 9x^6 - 8x^3 + 11$ with respect to x and evaluate it when $x = 1$.

solution: rate of change is the derivative

$$f'(x) = 9 \cdot 6x^5 - 8 \cdot 3x^2$$

$$f'(1) = 9 \cdot 6 \cdot 1^5 - 8 \cdot 3 \cdot 1^2$$
$$f'(1) = 54 - 24 = 30$$

(6) If a manufacturer's average cost function is given by

$$\overline{C} = .005q^2 - .5q + 70 + \frac{300}{q},$$

find the marginal cost function. What is the marginal cost when 50 units are produced.

solution: average cost means total cost divided by quantity: $\overline{C} = C/q$ and so

$$C = \overline{C} \cdot q = (.005q^2 - .5q + 70 + \frac{300}{q}) \cdot q = .005q^3 - .5q^2 + 70q + 300$$

marginal cost is the derivative of total cost, C.

$$C' = .005 \cdot 3q^2 - .5 \cdot 2q + 70$$

$q = 50 \Rightarrow$

$$C'(50) = .005 \cdot 3 \cdot 50^2 - .5 \cdot 2 \cdot 50 + 70 = 37.5 - 50 + 70 = 57.5$$