THINGS TO KNOW

This is by no means an exhaustive list of things to know and remember; this is a **minimum** amount of facts among those taught in class and that you are required to have learned.

Notations: x, y stand for variables and/or functions; f, g, h stand for functions; n stands for constants, usually integers; A, a, B, b stand for constants, numbers in general.

Any comments or corrections should be immediately directed to me:

cosmin@math.ohio-state.edu

Good luck!

(1) **algebra**

$$\frac{1}{x^n} = x^{-n}$$
$$\frac{1}{x^{-n}} = x^n$$
$$\ln(A \cdot B) = \ln(A) + \ln(B)$$
$$\ln(\frac{A}{B}) = \ln(A) - \ln(B)$$
$$\ln(A^B) = B \cdot \ln(A)$$

(2) **important derivatives**

$$(x^n)' = n \cdot x^{n-1}$$

$$(e^x)' = e^x$$

$$(a^x)' = a^x \cdot \ln(a)$$

$$(\ln(x))' = \frac{1}{x}$$

$$(\log_a(x))' = \frac{1}{x} \cdot \frac{1}{\ln(a)}$$

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(3) **important rules**

- Product Rule:

$$(f(x) \cdot g(x))' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

This form is PERFECTLY EQUIVALENT to any permutation, within the two terms, respectively (use whichever you feel like using ... but make sure it's a CORRECT version):

$$(f(x) \cdot g(x))' = g(x) \cdot f'(x) + f(x) \cdot g'(x)$$

$$(f(x) \cdot g(x))' = f'(x) \cdot g(x) + g'(x) \cdot f(x)$$

$$(f(x) \cdot g(x))' = g(x) \cdot f'(x) + g'(x) \cdot f(x)$$

I'm not recommending you use forms in which you first differentiate the second factor, g; it's easier to relate to the quotient rule if you use one of the above.

- Quotient Rule:

$$(\frac{f(x)}{g(x)})' = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{(g(x))^2}$$

This formula is also EQUIVALENT to a number of variations, due to the fact that multiplication is commutative ($A \cdot B = B \cdot A$) - one can switch within each of the terms the two factors:

$$(\frac{f(x)}{g(x)})' = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{(g(x))^2}$$

$$(\frac{f(x)}{g(x)})' = \frac{f'(x) \cdot g(x) - g'(x) \cdot f(x)}{(g(x))^2}$$

$$(\frac{f(x)}{g(x)})' = \frac{g(x) \cdot f'(x) - g'(x) \cdot f(x)}{(g(x))^2}$$

- Chain Rule:

Since we're only using it in a limited amount of situation, let's just analyze those cases only.

- Power Rule:

$$[(f(x))^n]' = n \cdot (f(x))^{n-1} \cdot f'(x)$$

- Exponential Rule:

$$(e^{f(x)})' = e^{f(x)} \cdot f'(x)$$

$$(a^{f(x)})' = a^{f(x)} \cdot \ln(a) \cdot f'(x)$$

- Logarithmic Rule:

$$[\ln(f(x)]' = \frac{1}{f(x)} \cdot f'(x)$$

$$[\log_a(f(x)]' = \frac{1}{f(x)} \cdot \frac{1}{\ln(a)} \cdot f'(x)$$

Be aware that you might see not only "simple" functions inside powers, exponentials and logarithms, but **products**, **quotients** of functions as well. In those cases, in the formulas above, the last factor ($f'(x)$) will involve using the appropriate rule for computation (product, quotient rule).

(4) **techniques for finding derivatives**

- Implicit Differentiation

  Any formula involving both the x and the y, and which cannot be reduced to a formula $y = f(x)$ can be differentiated as shown in the following example.

  $$x^3 - xy^2 + y^4 = 2x + y$$

  Differentiate both sides, append y' to each instance of "differentiating y" (that is, not ALL y will get that appendage, but those y involved in an actual differentiation).

  $$3x^2 - (1 \cdot y^2 + x \cdot 2y \cdot y') + 4y^3 \cdot y' = 2 + y'$$

  In the parenthesis above we used product rule; notice that y^2 does NOT have an y' appended to it, since there we only differentiate x; yet the $2y$ does have an y', since it's the result of differentiating y^2; similarly for $4y^3$, which is differentiated from y^4.

  $$3x^2 - y^2 - x \cdot 2y \cdot y' + 4y^3 \cdot y' = 2 + y'$$

  Cumulate all the terms involving y' in the left hand-side, and all the rest in the right hand-side.

  $$x \cdot 2y \cdot y' + 4y^3 \cdot y' - y' = 2 - 3x^2 + y^2$$

  Factor out the y'.

  $$(x \cdot 2y + 4y^3 - 1) \cdot y' = (2 - 3x^2 + y^2)$$

  $$y' = \frac{2 - 3x^2 + y^2}{x \cdot 2y + 4y^3 - 1}$$

  The formula for y' involves x as well as y; that's normal.

- Logarithmic Differentiation

$$f'(x) = [\ln(f(x))]' \cdot f(x)$$

The gist: we can compute the derivative of a function f by performing the following operations:
  - take the ln of the function (which will transform it into sums with coefficients, ergo simpler form)
  - differentiate the formula obtained
  - multiply the result of differentiation with the original f

(Look at the formula in page 3, logarithmic rule, and solve for $f'(x)$!!)
For any formula for a function $f(x)$ involving a large number of: products, powers, roots, computation of a derivative can be simplified using the logarithm (note: it's not needed for ALL functions; most simple formulas can easily be computed with the above means). Follow the example below.

$$f(x) = \frac{(x-1)^2(x-2)^3(x-3)^2}{(x+1)^3 \cdot \sqrt[3]{(x+2)}}$$

Since it's a pretty long formula, take the ln of f (**careful!**, NOT $\ln(x)$ TIMES $f(x)$!!):

$$\ln(f(x)) = \ln(\frac{(x-1)^2(x-2)^3(x-3)^2}{(x+1)^3 \cdot \sqrt[3]{(x+2)}}) =$$

$$= 2\ln(x-1) + 3\ln(x-2) + 2\ln(x-3) - 3\ln(x+1) - \frac{1}{3}\ln(x+2)$$

(products become sums, denominators are subtracted, powers become coefficients, roots become fractions; all at the expense of having some extra ln's involved)
Differentiate the formula above (refer to the third page, logarithmic rule):

$$[\ln(f(x))]' = [2\ln(x-1) + 3\ln(x-2) + 2\ln(x-3) - 3\ln(x+1) - \frac{1}{3}\ln(x+2)]' =$$

$$= 2 \cdot \frac{1}{x-1} \cdot (x-1)' + 3 \cdot \frac{1}{x-2} \cdot (x-2)' + 2 \cdot \frac{1}{x-3} \cdot (x-3)' - 3 \cdot \frac{1}{x+1} \cdot (x+1)' - \frac{1}{3} \cdot \frac{1}{x+2} \cdot (x+2)' =$$

$$= 2 \cdot \frac{1}{x-1} + 3 \cdot \frac{1}{x-2} + 2 \cdot \frac{1}{x-3} - 3 \cdot \frac{1}{x+1} - \frac{1}{3} \cdot \frac{1}{x+2}$$

Multiply by original f, hence getting the desired derivative:

$$f'(x) = \frac{(x-1)^2(x-2)^3(x-3)^2}{(x+1)^3 \cdot \sqrt[3]{(x+2)}} \cdot [2 \cdot \frac{1}{x-1} + 3 \cdot \frac{1}{x-2} + 2 \cdot \frac{1}{x-3} - 3 \cdot \frac{1}{x+1} - \frac{1}{3} \cdot \frac{1}{x+2}]$$