**THINGS TO KNOW**

This list comes as a completion of the list I designed for the second midterm. Again, it's not exhaustive, and contains a bare minimum.

Notations: $x$, $y$ stand for variables and/or functions; $f$, $g$, $h$ stand for functions; $n$ stands for constants, usually integers; $A$, $a$, $B$, $b$ stand for constants, numbers in general.

Any comments or corrections should be immediately directed to me:

**cosmin@math.ohio-state.edu**

Good luck!

(1) how to graph a function $f$

step 1a: compute first derivative, $f'$;

step 1b: find critical numbers (mostly, where $f'(x) = 0$; to be sure, check $f'(x) = DNE$, but since most of the functions are fractions, this case will give you numbers NOT in the domain);

step 1c: draw the table; settle on the signs between the critical numbers FOR THE FIRST DERIVATIVE (by plugging in some values, you choose them, and only looking at the sign)

step 2a: compute second derivative, $f''$;

step 2b: find $x$ so that $f''(x) = 0$;

step 2c: complete the table from step 1, by adding the new numbers obtained in 2b, and settling on the signs between these for the SECOND DERIVATIVE

step 3: draw (on the table; it's easiest) the shape of the graph, by combining the effects of increase/decrease with concavity up/down

step 4a: look for vertical asymptotes (ONLY FOR $f$ FRACTION!) by taking the values where the denominator is 0 (make sure you factor out the numerator and the denominator and cancel the common factors)

step 4b: look for horizontal asymptote (again, only for $f$ fraction) by taking the LIMIT to infinity; if power in the numerator is smaller or equal, you DO HAVE a horizontal asymptote (either at level 0, for smaller power, or at a non-zero level, for same power - ration of coefficients of biggest powers); if power in the numerator is bigger, no horizontal asymptote

step 5: "helper" points; find $y$-intercept; find $x$-intercepts, if obvious; compute values of $f$ in relative max and min (you can see those in the table!)

step 6: sketch the graph; first plot the helper points; then the asymptotes; then connect the helper points and asymptotes by curves following the shape from the table.

*Date*: March 8, 2003.

(2) how to find relative max, relative min for a function $f$

step 1: compute first derivative

step 2: find critical numbers: $f'(x) = 0$ or $f'(x) = DNE$ (the latter, VERY rare in our problems)

step 3: decide on the method (!); either first derivative test or second derivative test (the problem might REQUIRE you to use one or the other)

Assume First Derivative Test:

step 4-fdt: draw a table, put the critical numbers on top; find sign between these FOR THE DERIVATIVE, by plugging intermediate values, between those critical numbers

step 5-fdt: look for change of sign; $+$ to $-$ = increase, then decrease = rel max; $-$ to $+$ = decrease then increase = rel min; no change of sign, no relative extrema

Assume Second Derivative Test:

step 4-sdt: compute second derivative

step 5-sdt: plug into this second derivative the critical numbers; negative second derivative means concave down ($\frown$) = rel max; positive second derivative means concave up ($\smile$) = rel min; second derivative equal to 0 means NO INFORMATION (careful, it does not mean "neither", as in the First Derivative Test, simply means we cannot tell what it is by using this test).

(3) how to compute absolute max, absolute min

Remember, this is a COMPLETELY different problems than the one above, with the relative extrema!

Requirements: closed interval! (endpoints)
Step 1: compute derivative

Step 2: find critical numbers; select only those INSIDE the closed interval given (for example, if the interval given is $[0, 2]$ and one critical number is 3, ignore it.

Remark: you might be left with no critical numbers after checking which are inside the interval - that's OK.

Step 3: attach endpoints (see, even if you're left with no points after step 2, you get these 2 at least; also, sometimes one or both the endpoints are also critical numbers - can happpen)

Step 4: forget all about the derivative; you have a collection of numbers (critical and endpoints) and a function (the original function); plug in the numbers in the function, select the biggest output, and the smallest output, these are your ABSOLUTE MAX and MIN

(4) practical (word) problems

you will have to read the problem in order to see WHICH function needs to be maximized or minimized; not always will you have the function given, so you will need to produce it using the data at hand. Here are some relations that can help you:

- marginal cost/profit/revenue = derivative of TOTAL cost/profit/revenue
- revenue = quantity times price = $q \cdot p$
- profit = revenue minus cost
- demand is usually $p$ (price) in terms of $q$ (quantity); it's useful for computing REVENUE (see above) as we multiply the function that gives $p$ with an additional $q$, and we get revenue in terms of $q$, quantity

  Example: demand is $p = 150 - \frac{\sqrt{q}}{10}$; revenue is then

  $$r = p \cdot q = (150 - \frac{\sqrt{q}}{10}) \cdot q = 150q - \frac{q^{\frac{3}{2}}}{10}$$

- similarly, supply is $q$ quantity in terms of $p$ price; this time you will multiply the function that gives $q$ times an additional $p$ and get revenue in terms of $p$

(5) situations you can encounter in practical (word) problems

all the problems will ask you to maximize or minimize something; you absolutely-positively have to IDENTIFY the function that needs to be maximized/minimized ... circle it in the text, or write it somewhere in bold. next, you need to find the endpoints. here's what can happen

- the endpoints are explicitly given in the problem - good for us, we have all the elements for an ABSOLUTE MAX/MIN problem

- endpoints are not explicitly given, but there is enough information to obtain them; for example: quantities, prices are NEVER negative, and so an usual LEFT ENDPOINT IS ZERO. for the right endpoint look for information which bounds production/prices/profit/etc

- there is only information for one endpoint (the LEFT which is zero). for these problems you can assume that they either increase indefinitely, or decrease indefinitely on the right side. best bet is to "INVENT" an endpoint, the larger the better (it will not reduce the complexity of the problem). how you do it? after you make sure there is no way to produce the right endpoint, state: "I assume right endpoint to be 10000" (you can make it 1000, 100000, 1000000000, it's up to you; should be pretty large, however).

At this point you should have all the elements for an ABSOLUTE MAX/MIN problem - to solve it compute derivative, find critical numbers, attach endpoints, plug all those numbers back into the function (NOT THE DERIVATIVE!), select the biggest and/or smallest.