

FINAL EXAM

MATH 132 WINTER 2000

1. Compute the following limits (leave the answers in fractions)

(a) $\lim_{x \rightarrow 2} \frac{x^2 - 4x + 4}{x^2 + x - 6}$ (8 points)

(b) $\lim_{x \rightarrow -\infty} \frac{x^2 - 9x + 3}{x - 3 - 7^2}$ (8 points)

2. Find the derivatives of the following function (do not simplify)

(a) $f(t) = (5t^7 + 3t^2 - 5)(8t^5 - 5t + 2)$ (8 points)

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(b) $y = \frac{4 - x^3}{5x^2 + 7}$ (8 points)

(c) $f(x) = x^7 \ln(5x + 3)$ (8 points)

(d) $f(x) = \left(\frac{x-2}{x-7}\right)^7$ (8 points)

3. Let $f(x) = 11x^2 + 5$. Find

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

by finding $f(x + h)$, substitute and simplify. (8 points)

4. Let

$$f(x) = \begin{cases} \frac{5x-1}{2x-3}, & \text{if } x \leq 0 \\ \frac{7x}{9x-8}, & \text{if } x > 0 \end{cases}$$

Find:

(a) $\lim_{x \rightarrow 0^+} f(x)$ (4 points)

(b) $\lim_{x \rightarrow 0^-} f(x)$ (4 points)

4. did you notice that there are two “number 4” problems? weird :)

Find the equation of the tangent line to the graph of $y = -11x^3 + 9x + 8$ at the point $(0, 8)$ (8 points)

5. The total profit function for a product is given by

$$P(x) = 95 + 10x^2 - .5x^3$$

where x is the number of units sold.

(a) What function gives the marginal profit? (6 points)

(b) What is the marginal profit when 100 units are sold? (2 points)

6. Use derivatives to find absolute extrema of the function $y = f(x) = (x + 1)^3$ in the interval $[-2, 3]$ (8 points)

7. For the graph of the function $f(x) = x^3 + 3x^2 - 9x + 15$

(a) Find its y -intercept (3 points)

(b) use derivatives to find the interval(s) where $f(x)$ is increasing and where $f(x)$ is decreasing (5 points)

(c) Use information obtained in part (b) to find its point(s) of rel max and rel min (3 points)

(d) use derivatives to determine the interval(s) where it is concave up and where it is concave down (5 points)

(e) what are its point(s) of inflection? (3 points)

(f) sketch a graph of the function $f(x)$ using information obtained in parts (a)-(e) above (5 points)

8. Find $f(x)$ given $f'(x) = 7x^4 - 9x + 5$ and $f(0) = 5$ (8 points)

9. Evaluate the following integrals and simplify

(a) $\int \frac{5 dx}{9\sqrt{x^5}}$ (8 points)

(b) $\int \frac{\ln(x)}{x} dx$ (12 points)

10. Evaluate

$$\int_1^3 \frac{x^2 - 7x - 5}{x} dx \quad (8 \text{ points})$$

11. Evaluate (using tables) $\int \sqrt{4x^2 + 9} dx$. Write down the formula number you are using for this question (12 points)

12. Solve the inequality

$$\frac{(2x - 1)(3 - x)}{(5 + x)} \leq 0 \quad (8 \text{ points})$$

13. Find the area of the region bounded by $y = x^3 - x$ and the x -axis
(12 points)

14. Evaluate the integral

$$\int \frac{x + 1}{(x^2 + 2x + 2)^3} dx \quad (8 \text{ points})$$

15. If the demand equation for a product is given by $p = q^2 - 10q + 25$
and the supply equation is $p = q^2 + q + 3$

(a) find the equilibrium point

(b) find the consumer's surplus
(12 points)