SAMPLE MIDTERM II HINTS AND SOLUTIONS

1. For the function $f(x) = x^2 e^{-x}$

(a) use derivatives to determine the interval(s) (if any) on which $f(x)$ is increasing and the interval(s) (if any) on which $f(x)$ is decreasing (if there are none, please say so).

(b) Use the information obtained in part (a) to find the values of x for which $f(x)$ has relative max and relative min. (if there are none, please say so)).

2. Given that the points $(0,0)$, $(-1,-2)$, $(2,1.6)$, $(-2,-1.6)$, $(\sqrt{3}, \sqrt{3})$, $(-\sqrt{3}, -\sqrt{3})$ all lie on the graph of $y = f(x)$. From $f'(x)$ and $f''(x)$ find where f is decreasing, increasing, where f has rel max, rel. min, where f has points of inflection, where f is concave up, where f is concave down and sketch the graph roughly.

$$f'(x) = \frac{-4(x-1)(x+1)}{(x^2+1)^2}$$

$$f''(x) = \frac{8x(x-\sqrt{3})(x+\sqrt{3})}{(x^2+1)^3}$$

3. Find the critical points for $y = \frac{-1}{3} + 8x^2 - \frac{1}{6}x^6$

(a) classify them as rel max, rel min

(b) find its points of inflection

(c) sketch the graph and label the points

4. For a certain firm the total cost is given by

$$C(x) = 1200 + 12x + \frac{x^2}{2}$$

and

$$R(x) = 100x - \frac{x^2}{2}$$

is the total revenue.

Find the quantity that maximizes the profit, and find the maximum profit.

5. A farmer has 5000 feet of fence to enclose 3 sides of a rectangular field that has a river on one side. If all the fence is used, what dimensions will give a maximum area for the field?

6. Let

$$\text{(i) } f(x) = x^3 + 3x^2 - 9x + 25$$

$$\text{(ii) } f(x) = 4x^3 - 6x^2 - 24x + 25$$

(a) Find its y-intercept (for each, that is)

(b) Use derivatives to find the interval(s) where $f(x)$ is increasing and where $f(x)$ is decreasing.

(c) Use information obtained in part (b) to find its point of rel max and rel min

(d) Use derivatives to determine the interval(s) where it is concave up and where it is concave down

(e) What are its point(s) of inflection?

(f) sketch a graph of the function $f()$ showing all the information obtained in parts (a)-(e), labelling the points of rel max and rel min

7. (a) Use the second derivative test to find point(s) of rel. max and rel. min for the function
$$f(x) = x^3 - 3x^2 - 9x + 16$$

(b) Find the absolute max and absolute min for the function of part (a) in the interval $[-3, 3]$

8. For the functions
$$\text{(i)} \ \ f(x) = \frac{7x}{x+9}$$

$$\text{(ii)} \ \ f(x) = x(x-2)^3$$

(a) use derivatives to determine the interval(s) (if any) on which $f(x)$ is increasing and the interval(s) on which $f(x)$ is decreasing (if there are none, please say so)

(b) use the information obtained in part (a) to find relative max and relative min (if there are none, please say so)

9. Evaluate the following integrals:

(a) $\int (x^5 + x - 2) \, dx$

(b) $\int \frac{1}{7x^2} \, dx$

(c) $\int \frac{6x^4}{(3x^5)^3} \, dx$

(d) $\int \frac{x^4 - 3x^2 + 4x}{x} \, dx$

10. a) Given $f'(x) = 4x^3$ find $f(x)$; b) if in addition given $f(1) = 15$ determine C

11. a) Given $f'(x) = \frac{4}{x^3}$ find $f(x)$; b) if in addition given $f(2) = 8$ determine C

11 c. Given $f'(x) = 5\sqrt[3]{x^5} + 2$ and $f(0) = 1$ find $f(x)$

12. For the functions
$$\text{(i)} \ \ f(x) = \frac{7x}{x+9}$$

$$\text{(ii)} \ \ f(x) = x(x-2)^3$$

(a) use derivatives to determine the interval(s) (if any) on which $f(x)$ is increasing and the interval(s) on which $f(x)$ is decreasing (if there are none, please say so)

(b) use the information obtained in part (a) to find relative max and relative min (if there are none, please say so)

13. Find the derivatives of the following functions:

a) $y = x^2 7^{3x}$

b) $y = e^{x^3}$

c) $f(x) = \frac{xe^x}{x+1}$

d) $y = \ln(x\sqrt{3x-2})$

e) $f(x) = \ln(x^7 - 8x^5 + 7x - 3)$

h) $y = \ln(2x^2 - 3)$

i) $y = \frac{\ln(x)}{x}$