SOLUTIONS CHAPTER 12.3

MATH 132 WI01

6.

Proof. (a) $s(1) = -3 \cdot 1^2 + 2 \cdot 1 + 1 = -3 + 2 + 1 = 0$
(b) We also need $s(1.25) = -3 \cdot (1.25)^2 + 2 \cdot 1.25 + 1 = -4.6875 + 2.5 + 1 = -1.1875$; **average velocity** is computed dividing difference in positions (that is, distance travelled) by difference in times (that is, time elapsed):

$$\text{average velocity} = \frac{s(1.25) - s(1)}{1.25 - 1} = \frac{-1.1875 - 0}{.25} = \frac{-1.1875}{0.25} = -4.75$$

(c) For **velocity at a given t-value** we need the derivative $s' = (-3) \cdot 2t + 2 = -6t + 2$ - and we just plug in $t = 1$:

$$\text{velocity at } t = 1 = -6 \cdot 1 + 2 = -6 + 2 = -4$$

□

12.

Proof. The problem asks **rate of change of A with respect to r**! this can be translated as **derivative of A**! simple as that.

$$A' = \pi \cdot 2r = 2\pi r$$

and plug in $r = 3$

$$A'(3) = 2\pi \cdot 3 = 6\pi$$

□

22.

Proof. Since $\overline{c}$ is the **average cost**, the total cost c equals $\overline{c} \cdot q = (2 + \frac{1000}{q}) \cdot q = 2q + 1000$. Marginal cost is **the derivative of** c, so it's

$$c' = 2$$

in which we have to plug in $q = 235$; but c' is constant! so anything we plug in it gives us 2! so the result is

$$c'(235) = 2$$

□

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34.

Proof. Again, notice the formulation of the question:**rate of change of** c **with respect to** q=**derivative of** c!

$$c' = 0.4 \cdot 2q + 4 = 0.8q + 4$$

in which we plug in $q = 2$ and we get

$$c'(2) = 0.8 \cdot 2 + 4 = 1.6 + 4 = 5.6$$

As for $\frac{\Delta c}{\Delta q}$ we **don't use the derivative at all!**; what we need is

$$\Delta c = c(3) - c(2) = (0.4 \cdot 3^2 + 4 \cdot 3 + 5) - (0.4 \cdot 2^2 + 5 \cdot 2 + 5) =$$
$$= 3.6 + 12 + 5 - 1.6 - 10 - 5 = 4$$
$$\Delta q = 3 - 2 = 1$$

and so

$$\frac{\Delta c}{\Delta q} = \frac{4}{1} = 4$$

□

36.

Proof. (a) rate of change of y with respect to x=**derivative of** y=$y' = f'(x) = (4 - 2x)' = -2$
(b) relative rate of change=$\frac{y'}{y} = \frac{-2}{4-2x}$
(c) plug in $x = 3$ in the formula in (a): $y'(3) = -2$ (it's a constant! so anything we plug in will give us -2)
(d) plug in $x = 3$ in the formula in (b): $\frac{y'}{y}(3) = \frac{-2}{4-2\cdot 3} = \frac{-2}{4-6} = \frac{-2}{-2} = 1$
(e) just express the result in (d) in percentage form: $1 = \frac{100}{100}\% = 100\%$ □