SOLUTIONS CHAPTER 12.6

MATH 132 WI01

4.

Proof.

$$\frac{dy}{dx} = \frac{dy}{dz}\frac{dz}{dx}$$

but

$$\frac{dy}{dz} = (z^{\frac{1}{3}})' = \frac{1}{3}z^{\frac{1}{3}-1} = \frac{1}{3}z^{-\frac{2}{3}}$$

$$\frac{dz}{dx} = (x^6 - x^2 + 1)' = 6x^5 - 2x$$

We have to multiply these two, but since we don't want to have z in the final formula, we'll replace every occurence of z by its formula, $z = x^6 - x^2 + 1$ (BEWARE! not the formula for the DERIVATIVE of z!!!!):

$$\frac{dy}{dx} = \frac{1}{3}z^{-\frac{2}{3}}(6x^5 - 2x) = \frac{1}{3}(x^6 - x^2 + 1)^{-\frac{2}{3}}(6x^5 - 2x)$$

□

8.

Proof. First find $\frac{dy}{dx}$, using the above method:

$$\frac{dy}{du} = 9u^2 - 2u + 7$$

$$\frac{du}{dx} = 3$$

and hence

$$\frac{dy}{dx} = (9u^2 - 2u + 7)(3) = [9(3x-2)^2 - 2(3x-2) + 7](3)$$

Now plug in $x = 1$; we get

$$\frac{dy}{dx}(1) = [9(3-2)^2 - 2(3-2) + 7](3) = 14 \cdot 3 = 42$$

□

Date: 01/21/2000.

22.

Proof.

$$y = \sqrt[3]{8x^2 - 1} = (8x^2 - 1)^{\frac{1}{3}}$$

so then, using **Chain Rule**, we get

$$y' = \frac{1}{3}(8x^2 - 1)^{\frac{1}{3}-1}(8x^2 - 1)' = \frac{1}{3}(8x^2 - 1)^{-\frac{2}{3}} \cdot 16x$$

□

28.

Proof.

$$y = \frac{1}{(1-x)^3} = (1-x)^{-3}$$

so then, using **Chain Rule**, we get

$$y' = (-3)(1-x)^{-3-1}(1-x)' = -3(1-x)^{-4}(-1)$$

□

46.

Proof. Use **Quotient Rule**:

$$y' = \frac{[(2x+3)^3]'(x^2+4) - (2x+3)^3(x^2+4)'}{(x^2+4)^2} =$$

$$= \frac{[3(2x+3)^2(2x+3)'](x^2+4) - (x^2+4)^3(2x)}{(x^2+4)^2} =$$

$$= \frac{[3(2x+3)^2(2)](x^2+4) - (x^2+4)^3(2x)}{(x^2+4)^2}$$

□

62.

Proof. First we need y':

$$y = \frac{-3}{(3x^2+1)^3} = (-3) \cdot (3x^2+1)^{-3}$$

$$y' = (-3)[(-3)(3x^2+1)^{-3-1}(3x^2+1)'] = (-3)[(-3)(3x^2+1)^{-4}(6x)]$$

Simplify:

$$y' = \frac{54}{(3x^2+1)^4}$$

Now we can find the **slope of the tangent line** by plugging in $x = 0$:

$$\text{slope} = \frac{54}{(3 \cdot 0^2 + 1)^4} = \frac{54}{1^4} = 54$$

Having the slope and the point $(0, -3)$ we get for the **equation of the tangent line using point-slope formula**

$$y - (-3) = 54(x - 0)$$

$\square$