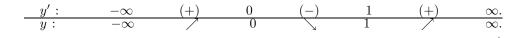
SOLUTIONS CHAPTER 14.1

MATH 132 WI01

2. Function is "\" decreasing in $(-\infty,-1)$, then "\" increasing in (-1,0), again "\" decreasing in (0,1), and finally "\" increasing in $(1,\infty)$. Relative extrema are points where the function's behaviour changes from increasing to decreasing - maximum - which in our case is (0,0), or from decreasing to increasing - minimum - which in our case are (-1,-1) and (1,-1).

6. What we are interested in is the sign of the derivative, derivative which is already given in our case. Hence, let's look for sign of $f'(x) = 2x(x-1)^3$. f'(x) equals zero in x = 0 and in x = 1, so we have to check sign between these values, and outside these values: plug in -1, we get $2(-1)(-2)^3 = 16$, hence positive, hence the original f is **increasing in the interval** $(-\infty,0)$; plug in $\frac{1}{2}$, we get $2*(\frac{1}{2})(\frac{1}{2}-1)^3 = (-\frac{1}{2})^3 = -\frac{1}{8}$, hence negative, so the original f function will be **decreasing in the interval** (0,1); plug in 2, we get $2*2(2-1)^3 = 4$, hence again positive, so now the function f is again **increasing in the interval** $(1,\infty)$. As you can see, at 0 we have a change from increasing to decreasing, so it's a relative **maximum**, and at 1 we have a change from decreasing to increasing, so it's a relative **minimum**.



24. $y=5x-x^5$. First compute the derivative: $y'=5-5x^4$. Now find zeroes of y': $5-5x^4=0 \to 1-x^4=0 \to 1=x^4 \to x=\pm 1$. So we have to check sign of y' in $(-\infty,-1)$: plug in -2 in y', $5-5(-2)^4=5-80=-75$, negative, so y is **decreasing in** $(-\infty,-1)$. Check sign of y' in (-1,1): plug in 0 in y', $5-5*0^4=5$, positive, so y is **increasing in** (-1,1). Check sign of y' in $(1,\infty)$: plug in 2 in y', $5-5*2^4=5-80=-75$, negative, so y is **decreasing in** $(1,\infty)$. As for relative extrema, we look at the critical numbers, hence -1 and +1: at -1, y changes from decreasing to increasing, so it's a relative **minimum**, and at +1, y changes from increasing to decreasing, so it's a relative **maximum**.

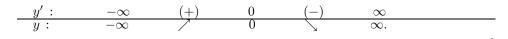
38. $y = \frac{x^2}{x^2 - 9}$. Again, let's compute derivative of y, using **Quotient Rule**:

$$y' = \frac{2x * (x^2 - 9) - x^2 * 2x}{(x^2 - 9)^2} = \frac{2x^3 - 18x - 2x^3}{(x^2 - 9)^2} = \frac{-18x}{(x^2 - 9)^2}$$

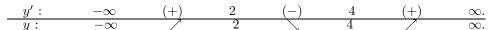
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Zero for this function is only for x=0, but there's also a denominator involved, so we also have points where the derivative is NOT DEFINED, namely at $x=\pm 3$. But we are in luck! since for $x=\pm 3$ y itself is not defined. So the only critical number is x=0. Now about the sign: a good starting point is to notice that the denominator for y' is a square! so it's always positive, so when we check the sign of y' we just need to check the sign of the numerator (a positive denominator doesn't change the sign of the fraction); so we just need to see what happens in the interval $(-\infty,0)$, which we check by pluging in -1 in the numerator, giving us -18*(-1)=18, positive, hence y is **increasing** on this interval, and we need to see what happens in the interval $(0,\infty)$, by pluging in 1 in the numerator, which gives us -18*1=-18, negative, hence y is **decreasing** in this interval. As for relative extrema, we're just interested in 0, and in 0, y changes from increasing to decreasing, so 0 is a relative **maximum**.



58. $y = x^3 - 9x^2 + 24x - 19$. As before, let's compute the derivative: $y' = 3x^2 - 9 * 2x + 24 = 3x^2 - 18x + 24 = 3(x^2 - 6x + 8) = 3(x - 2)(x - 4)$. We notice that the critical numbers, that is the zeroes for y' are 2 and 4. So we are interested in seeing what is happening outside the roots, and in between.For $(-\infty, 2)$ let's plug in 0 in y': 3*(-2)*(-4) = 24, positive, so y is **increasing** in this interval. For (2,4) let's plug in 3 in y': 3*1*(-1) = -3, negative, so y in **decreasing** in this interval. Lastly, for $(4,\infty)$ let's plug in 5 in y': 3*3*1 = 9, positive, so y is **increasing** again. As for relative extrema, let's see what happens to the two critical numbers: in 2, y changes from increasing to decreasing, hence it's a relative **maximum**; in 4, y changes from decreasing to increasing, so it's a relative **minimum**.



What we are interested in doing now is finding a few reference values of y, that will help localize the graph of y. One intercept that can be obtained easily is the y-intercept, namely $y(0) = 0^3 - 9*0^2 + 24*0 - 19 = -19$. Then, we need the coordinates for the relative extrema: $y(2) = 2^3 - 9*2^2 + 24*2 - 19 = 8 - 36 + 48 - 19 = 1$ and $y(4) = 4^3 - 9*4^2 + 24*4 - 19 = 64 - 144 + 96 - 19 = -3$. Let's sketch the graph.