SOLUTIONS CHAPTER 14.1

MATH 132 WI01

2. Function is $'' \searrow''$ **decreasing** in ($-\infty$,-1), then $'' \nearrow''$ **increasing** in (-1,0), again $'' \searrow''$ **decreasing** in (0,1), and finally $'' \nearrow''$ **increasing** in (1,∞). Relative extrema are points where the function's behaviour changes from increasing to decreasing - **maximum** - which in our case is $(0,0)$, or from decreasing to increasing - **minimum** - which in our case are $(-1,-1)$ and $(1,-1)$.

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6. What we are interested in is the sign of the derivative, derivative which is already given in our case. Hence, let's look for sign of $f'(x) = 2x(x-1)^3$. $f'(x)$ equals zero in $x = 0$ and in $x = 1$, so we have to check sign between these values, and outside these values: plug in -1, we get $2(-1)(-2)^3 = 16$, hence positive, hence the original f is **increasing in the interval** $(-\infty, 0)$; plug in $\frac{1}{2}$, we get $2 * (\frac{1}{2})(\frac{1}{2} - 1)^3 = (-\frac{1}{2})^3 = -\frac{1}{8}$, hence negative, so the original f function will be **decreasing in the interval** $(0,1)$; plug in 2, we get $2*2(2-1)^3 = 4$, hence again positive, so now the function f is again **increasing in the interval** $(1,\infty)$. As you can see, at 0 we have a change from increasing to decreasing, so it's a relative **maximum**, and at 1 we have a change from decreasing to increasing, so it's a relative **minimum**.

y' :	$-\infty$	$(+)$	0	$(-)$	1	$(+)$	∞.
y :	$-\infty$	$\nearrow$	0	$\searrow$	1	$\nearrow$	∞.

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24. $y = 5x - x^5$. First compute the derivative: $y' = 5 - 5x^4$. Now find zeroes of y': $5 - 5x^4 = 0 \to 1 - x^4 = 0 \to 1 = x^4 \to x = \pm 1$. So we have to check sign of y' in $(-\infty, -1)$: plug in -2 in y', $5 - 5(-2)^4 = 5 - 80 = -75$, negative, so y is **decreasing in** $(-\infty, -1)$. Check sign of y' in $(-1, 1)$: plug in 0 in y', $5 - 5*0^4 = 5$, positive, so y is **increasing in** $(-1, 1)$. Check sign of y' in $(1, \infty)$: plug in 2 in y', $5 - 5*2^4 = 5 - 80 = -75$, negative, so y is **decreasing in** $(1, \infty)$. As for relative extrema, we look at the critical numbers, hence -1 and $+1$: at -1, y changes from decreasing to increasing, so it's a relative **minimum**, and at $+1$, y changes from increasing to decreasing, so it's a relative **maximum**.

y' :	$-\infty$	$(-)$	-1	$(+)$	1	$(-)$	∞.
y :	$-\infty$	$\searrow$	-1	$\nearrow$	1	$\searrow$	∞.

38. $y = \frac{x^2}{x^2-9}$. Again, let's compute derivative of y, using **Quotient Rule**:

$$y' = \frac{2x*(x^2-9) - x^2 * 2x}{(x^2-9)^2} = \frac{2x^3 - 18x - 2x^3}{(x^2-9)^2} = \frac{-18x}{(x^2-9)^2}$$

Date: 02/05/2001.

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Zero for this function is only for $x = 0$, but there's also a denominator involved, so we also have points where the derivative is NOT DEFINED, namely at $x = \pm 3$. But we are in luck! since for $x = \pm 3$ y itself is not defined. So the only critical number is $x = 0$. Now about the sign: a good starting point is to notice that the denominator for y' is a square! so it's always positive, so when we check the sign of y' we just need to check the sign of the numerator (a positive denominator doesn't change the sign of the fraction); so we just need to see what happens in the interval $(-\infty, 0)$, which we check by pluging in -1 in the numerator, giving us $-18 * (-1) = 18$, positive, hence y is **increasing** on this interval, and we need to see what happens in the interval $(0, \infty)$, by pluging in 1 in the numerator, which gives us $-18 * 1 = -18$, negative, hence y is **decreasing** in this interval. As for relative extrema, we're just interested in 0, and in 0, y changes from increasing to decreasing, so 0 is a relative **maximum**.

y' :	$-\infty$	(+)	0	(−)	∞
y :	$-\infty$	$\nearrow$	0	$\searrow$	∞.

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58. $y = x^3 - 9x^2 + 24x - 19$. As before, let's compute the derivative: $y' = 3x^2 - 9 * 2x + 24 = 3x^2 - 18x + 24 = 3(x^2 - 6x + 8) = 3(x - 2)(x - 4)$. We notice that the critical numbers, that is the zeroes for y' are 2 and 4. So we are interested in seeing what is happening outside the roots, and in between.For $(-\infty, 2)$ let's plug in 0 in y': $3 * (-2) * (-4) = 24$, positive, so y is **increasing** in this interval. For $(2, 4)$ let's plug in 3 in y': $3 * 1 * (-1) = -3$, negative, so y in **decreasing** in this interval. Lastly, for $(4, \infty)$ let's plug in 5 in y': $3 * 3 * 1 = 9$, positive, so y is **increasing** again. As for relative extrema, let's see what happens to the two critical numbers: in 2, y changes from increasing to decreasing, hence it's a relative **maximum**; in 4, y changes from decreasing to increasing, so it's a relative **minimum**.

y' :	$-\infty$	(+)	2	(−)	4	(+)	∞.
y :	$-\infty$	$\nearrow$	2	$\searrow$	4	$\nearrow$	∞.

What we are interested in doing now is finding a few reference values of y, that will help localize the graph of y. One intercept that can be obtained easily is the y-intercept, namely $y(0) = 0^3 - 9 * 0^2 + 24 * 0 - 19 = -19$. Then, we need the coordinates for the relative extrema: $y(2) = 2^3 - 9 * 2^2 + 24 * 2 - 19 = 8 - 36 + 48 - 19 = 1$ and $y(4) = 4^3 - 9 * 4^2 + 24 * 4 - 19 = 64 - 144 + 96 - 19 = -3$. Let's sketch the graph.