SOLUTIONS CHAPTER 14.2

MATH 132 WI01

2. $f(x) = -2x^2 - 6x + 5$, and the interval of definition is $[-2, 3]$. To find **absolute** extrema we are intersted in finding a few good numbers, as few as possible, and plug those numbers in f, and compare the results - by picking the largest result we want to get the **absolute maximum**, and by picking the smallest result we want to have the **absolute minimum**. How do we find those numbers? Two candidates are the definition interval's **endpoints** - in our case -2 and 3. The others are going to be the **critical numbers**, that is, zeroes and DNEs [1] of the derivative of f, f'. Let's compute f': $f'(x) = -2 * 2x - 6 = -4x - 6$. $f'(x) = 0 \rightarrow -4x - 6 = 0 \rightarrow -4x = 6 \rightarrow x = \frac{6}{(-4)} \rightarrow x = -\frac{3}{2}$. f' exists for every x, so there's no case of DNE, so $-\frac{3}{2}$ is the only critical number. The last thing we must make sure is whether the critical number is INSIDE the interval $[-2, 3]$ ... well, in our case it is! Let's start pluging in numbers:

$$f(-2) = -2 * (-2)^2 - 6 * (-2) + 5 = -2 * 4 + 12 + 5 = -8 + 12 + 5 = 9$$
$$f(3) = -2 * 3^2 - 6 * 3 + 5 = -2 * 9 - 18 + 5 = -18 - 18 + 5 = -27$$
$$f(-\tfrac{3}{2}) = -2*(-\tfrac{3}{2})^2 - 6*(-\tfrac{3}{2}) + 5 = -2*\tfrac{9}{4} + 3*3 + 5 = -\tfrac{9}{2} + 9 + 5 = -4.5 + 14 = 9.5$$

Biggest value is 9.5 which means that $-\frac{3}{2}$ is our **absolute maximum**, and smallest value is -27, which implies that 3 is the **absolute** looser ... uh, **minimum**.

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8. Same principle: first, derivative of $f(x)$.

$$f'(x) = (\frac{7}{3}x^3 + 2x^2 - 3x + 1)' = \frac{7}{3} * 3x^2 + 2 * 2x - 3 = 7x^2 + 4x - 3 = (7x - 3)(x + 1)$$

Critical numbers are, therefore, what we get from $7x - 3 = 0 \rightarrow 7x = 3 \rightarrow x = \frac{3}{7}$ and $x + 1 = 0 \rightarrow x = -1$. Check now against the interval: $\frac{3}{7}$ IS inside $[0, 3]$, but -1 is not (it's negative, so it cannot be to the right of 0). So we ignore -1, and we consider only $\frac{3}{7}$ and the endpoints of our interval, 0 and 3. Let's plug these numbers in f:

$$f(0) = \tfrac{7}{3} * 0^3 + 2 * 0^2 - 3 * 0 + 1 = 1$$
$$f(3) = \tfrac{7}{3} * 3^3 + 2 * 3^2 - 3 * 3 + 1 = 63 + 18 - 9 + 1 = 73$$
$$f(\tfrac{3}{7}) = \tfrac{7}{3} * (\tfrac{3}{7})^3 + 2 * (\tfrac{3}{7})^2 - 3 * \tfrac{3}{7} + 1 = \tfrac{9}{49} + \tfrac{18}{49} - \tfrac{9}{7} + 1 = \tfrac{27}{49} - \tfrac{63}{49} + \tfrac{49}{49} = \tfrac{13}{49} = 0.2653$$

Biggest value is 73, hence **absolute maximum** is for $x = 3$, smallest value is 0.2653, hence **absolute minimum** is for $x = \frac{3}{7}$.

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[1] DNE = Does Not Exist $\rightarrow$ values of x for which $f(x)$ is NOT defined - as an example, $x = 0$ for the function $\frac{1}{x}$

12. $f(x) = \frac{x}{x^2+1}$, and the interval is $[0, 2]$. Derivative of f (**quotient rule**):

$$f'(x) = \frac{1 * (x^2 + 1) - x * 2x}{(x^2 + 1)^2} = \frac{x^2 + 1 - 2x^2}{(x^2 + 1)^2} = \frac{1 - x^2}{(x^2 + 1)^2}$$

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The denominator is never 0 ($x^2 + 1 \geq 1$), so no DNEs; critical numbers will be produced only by the numerator: $1 - x^2 = 0 \rightarrow 1 = x^2 \rightarrow x = \pm 1$. Which of them is inside the interval? -1 is not in $[0, 2]$, but 1 is. So ... ignore -1, and let's plug 0, 2 (endpoints) and 1 (critical number) in f:

$f(0) = \frac{0}{0^2+1} = 0$
$f(2) = \frac{2}{2^2+1} = \frac{2}{4+1} = \frac{2}{5} = 0.4$
$f(1) = \frac{1}{1^2+1} = \frac{1}{2} = 0.5$

Biggest value is 0.5, so absolute maximum is in $x = 1$, and smallest value is 0, so absolute minimum is in $x = 0$.

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