SOLUTIONS CHAPTER 14.4

MATH 132 WI01

2. $y = -2x^2 + 6x + 12$. Find first critical numbers, by means of finding zeroes of first derivative: $y' = -2 * 2x + 6 = -4x + 6 = 0 \to 6 = 4x \to x = \frac{6}{4} = 1.5$. To check now whether it is or not maximum or minimum, we plug it in the second derivative $y'' = -4$: $y''(1.5) = -4 \to$ *negative* $\to (\frown) \to 1.5$ is a relative **maximum**.

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5. $y = x^3 - 27x + 1$. $y' = 0 \to 3x^2 - 27 = 0 \to 3x^2 = 27 \to x^2 = 9 \to x = \pm 3$. Plug these values in the second derivative $y'' = 3 * 2x = 6x$: $y''(-3) = 6 * (-3) = -18 \to$ *negative* $\to (\frown) \to -3$ is a relative **maximum**; $y''(3) = 6 * 3 = 18 \to$ *positive* $\to (\smile) \to 3$ is a relative **minimum**.

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7. $y = -x^3 + 3x^2 + 1$. $y' = 0 \to -3x^2 + 3 * 2x = 0 \to 3x(-x + 2) = 0 \to x = 0, -x + 2 = 0 \to x = 2$. Plug these values in second derivative $y'' = -3 * 2x + 3 * 2 = 6(-x + 1)$: $y''(0) = 6(-0 + 1) = 6 \to$ *positive* $\to (\smile) \to 0$ is a relative **minimum**; $y''(2) = 6(-2+1) = 6*(-1) = -6 \to$ *negative* $\to (\frown) \to 2$ is a relative **maximum**.

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11. $y = 81x^5 - 5x$. $y' = 0 \to 81 * 5x^4 - 5 = 0 \to 81 * 5x^4 = 5 \to x^4 = \frac{1}{81} \to x = \pm \frac{1}{3} = \pm 0.33$. Plug these values in the second derivative $y'' = 81 * 5 * 4x^3 = 1620x^3$: $y''(-0.33) = -60 \to$ *negative* $\to (\frown) \to -0.33$ is a relative maximum; $y''(0.33) = 60 \to$ *positive* $\to (\smile) \to 0.33$ is a relative minimum.

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13. $y = (x^2 + 7x + 10)^2$. $y' = 0 \to 2(x^2 + 7x + 10) * (2x + 7) = 0 \to 2 * (x + 2)(x + 5)(2x + 7) = 0 \to x + 2 = 0, x = -2; x + 5 = 0, x = -5; 2x + 7 = 0, 2x = -7, x = -\frac{7}{2} = -3.5$. Plug these values in the second derivative $y'' = [2(x + 2)(x + 5)(2x + 7)]' = 2[(x^2 + 7x + 10)(2x + 7)]' = 2[(2x + 7)(2x + 7) + (x^2 + 7x + 10) * 2] = 2(4x^2 + 28x + 49 + 2x^2 + 14x + 20) = 2(6x^2 + 42x + 69)$:

$y''(-2) = 2(6*(-2)^2 + 42(-2) + 69) = 2(24 - 84 + 69) = 18 \to$ *positive* $\to (\smile) \to$ x=-2 is a relative minimum;

$y''(-5) = 2(6(-5)^2 + 42(-5) + 69) = 2(150 - 210 + 69) = 18 \to$ *positive* $\to (\smile) \to$ x=-5 is also a relative minimum;

$y''(-3.5) = 2(6(-3.5)^2 + 42(-3.5) + 69) = -9 \to$ *negative* $\to (\frown) \to$ x=-3.5 is a relative maximum.

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Date: 02/06/2001.