SOLUTIONS CHAPTER 16.3

MATH 132 - WI00

4. $\int(3x^2+14x)(x^3+7x^2+1)\,dx$. Having two polinomials, take the one of bigger degree, in our case x^3+7x^2+1, to be u and see if one can use it as the new variable:

$$u = x^3 + 7x^2 + 1$$

Find du by differentiating the formula in x:

$$du = (x^3 + 7x^2 + 1)'\,dx = (3x^2 + 7 * 2x)\,dx = (3x^2 + 14x)\,dx$$

Solve for dx:

$$dx = \frac{du}{3x^2 + 14x}$$

Try to use it in the integral:

$$\int (3x^2 + 14x)(x^3 + 7x^2 + 1)\,dx$$

$$\int (3x^2 + 14x)u\frac{du}{3x^2 + 14x}$$

$$\int u\,du = \frac{u^2}{2} + C = \frac{(x^3 + 7x^2 + 1)^2}{2} + C$$

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14.

$$\int x\sqrt{1 + 2x^2}\,dx$$

Again, two polinomials (one is x and the other one is $1+2x^2$), so choose the one of higher degree to be $u \to u = 1 + 2x^2$. Let's see if it fits:

$$du = (1 + 2x^2)'\,dx = (0 + 2 * 2x)\,dx = 4x\,dx$$

Solve for dx:

$$dx = \frac{du}{4x}$$

$$\int x\sqrt{1 + 2x^2}\,dx = \int x\sqrt{u}\,\frac{du}{4x}$$

$$\int \sqrt{u}\frac{1}{4}\,du = \frac{1}{4}\int u^{\frac{1}{2}}\,du$$

$$\frac{1}{4}\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{1}{4}\frac{(1 + 2x^2)^{\frac{3}{2}}}{\frac{3}{2}} + C$$

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26.

$$\int \frac{2x + 1}{x + x^2}\,dx$$

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Highest degree? The polinomial $x + x^2$. Take then

$$u = x + x^2$$

$$du = (x + x^2)'\,dx = (1 + 2x)\,dx$$

Solve for dx:

$$dx = \frac{du}{1 + 2x}$$

Plug these in the integral:

$$\int \frac{1 + 2x}{x + x^2}\,dx = \int \frac{1 + 2x}{u}\,\frac{du}{1 + 2x}$$

$$\int \frac{1}{u}\,du = \ln(u) + C = \ln(x + x^2) + C$$

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44.

$$\int \frac{x^2}{\sqrt[3]{2x^3 + 9}}\,dx$$

Highest degree polinomial is $2x^3 + 9$, so take

$$u = 2x^3 + 9$$

$$du = (2x^3 + 9)'\,dx = (2 * 3x^2 + 0)\,dx = 6x^2\,dx$$

Solve for dx:

$$dx = \frac{du}{6x^2}$$

Plug these in the integral:

$$\int \frac{x^2}{\sqrt[3]{2x^3 + 9}}\,dx = \int \frac{x^2}{\sqrt[3]{u}}\,\frac{du}{6x^2}$$

$$\int \frac{1}{u^{\frac{1}{3}}}\,\frac{1}{6}\,du = \frac{1}{6}\int u^{-\frac{1}{3}}\,du = \frac{1}{6}\,\frac{u^{1-\frac{1}{3}}}{1 - \frac{1}{3}} + C$$

$$\frac{1}{6}\,\frac{u^{\frac{2}{3}}}{\frac{2}{3}} + C = \frac{1}{6}\,\frac{(2x^3 + 9)^{\frac{2}{3}}}{\frac{2}{3}} + C$$

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56.

$$\int \frac{3}{7}(v - 2)e^{2-4v+v^2}\,dv$$

Again, look for polinomials - $2 - 4v + v^2$ would be worth a try:

$$u = 2 - 4v + v^2$$

$$du = (2 - 4v + v^2)'\,dv = (-4 + 2v)\,dv = 2(-2 + v)\,dv = 2(v - 2)\,dv$$

Solve for dv:

$$dv = \frac{du}{2(v - 2)}$$

Plug these in the integral:

$$\int \frac{3}{7}(v - 2)e^{2-4v+v^2}\,dv = \int \frac{3}{7}(v - 2)e^{u}\,\frac{du}{2(v - 2)}$$

$$\int \frac{3}{7}e^u \frac{1}{2}\,du = \frac{3}{7}\frac{1}{2}\int e^u\,du = \frac{3}{14}e^u + C = \frac{3}{14}e^{2-4v+v^2} + C$$

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78.

$$\int \frac{1}{t^2}\sqrt{\frac{1}{t}-1}\,dt$$

No more polinomials this time, but let's do the following trick, namely write everything (fractions, that is) as powers of t:

$$\int t^{-2}\sqrt{t^{-1}-1}\,dt$$

Which of these two expressions has higher power? -2 is **less** than -1, so try:

$$u = t^{-1} - 1$$

$$du = (t^{-1}-1)'\,dt = ((-1)t^{-2} - 0)\,dt = -t^{-2}\,dt$$

Solve for dt:

$$dt = \frac{du}{-t^{-2}}$$

Plug these in the integral:

$$\int t^{-2}\sqrt{t^{-1}-1}\,dt = \int t^{-2}\sqrt{u}\,\frac{du}{-t^{-2}}$$

$$\int \sqrt{u}\frac{1}{-1}\,du = \frac{1}{-1}\int u^{\frac{1}{2}}\,du$$

$$\frac{1}{-1}\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{1}{-1}\frac{(t^{-1}-1)^{\frac{3}{2}}}{\frac{3}{2}} + C$$

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