SOLUTIONS FOR HOMEWORK PROBLEMS 16.9

MATH 132 - WI00

$\boxed{4}$

Let's forget about the graphs ... first thing, where do these functions intersect? $x = x(x-2)^2 \to x - x(x-2)^2 = 0 \to x[1-(x-2)^2] = 0 \to x[(1-(x-2))(1+(x-2))] = 0$ (difference of squares, where $1 = 1^2$ - right?). $x(3-x)(x-1) = 0 \to x = 0, x = 1, x = 3$.

So we have 3 intersection points, which forces us to compute the integral on two intervals separately, and "add" the results in the very end (in the sense that we ignore the sign of whatever we get there, and add only the positive values together).

$\int_0^1 [x - x(x-2)^2]\,dx = \int_0^1 [x - x(x^2 - 4x + 4)]\,dx = \int_0^1 (x - x^3 + 4x^2 - 4x)\,dx = \int_0^1 (-x^3 + 4x^2 - 3x)\,dx = (-\frac{x^4}{4} + 4\frac{x^3}{3} - 3\frac{x^2}{2})|_0^1 = (-\frac{1}{4} + 4\frac{1}{3} - 3\frac{1}{2}) - 0 =$ something

$\int_1^3 [x - x(x-2)^2]\,dx = ... = (-\frac{x^4}{4} + 4\frac{x^3}{3} - 3\frac{x^2}{2})|_1^3 =$ something

Now ... if you want to use the graph, it will tell you that the result you get for the first integral is negative, and for the second it's positive ... since x is below $x(x-2)^2$ in the first interval, and above in the second.

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First, since we have a formula which involves y^2 (which usually is bad to just try and get rid of), let's express x in the second equation in terms of y too, namely $x = 2 - y$. Since now we are given functions in the reversed order of variables (namely $x = y^2$ and $y = 2 - x$) we should (if not in writing, then mentally) think x is actually y and y is actually x (like, the functions were then $y = x^2$ and $y = 2 - x$, right? it's truly just a change of order of variables). So, let's proceed in finding the intersection points: $y^2 = 2 - y \to y^2 + y - 2 = 0 \to (y-1)(y+2) = 0 \to y = 1$ and $y = -2 \to x = y^2 = 1^2 = 1$ and $x = y^2 = (-2)^2 = 4$. What does "fourth quadrant" mean? it's where x is positive and y is negative. But (1,1) is not in there! so only (4, -2) remains. But then, since we have only one intersection point, what do we do? we take the other limit the problem (in a hidden way, true) takes, namely 0 ("fourth quadrant" is bounded by 0 on the y-axis). We need to compute now $\int_{-2}^0 (y^2 - (2-y))\,dy = \int_{-2}^0 (y^2 + y - 2)\,dy = \frac{y^3}{3} + \frac{y^2}{2} - 2y|_{-2}^0 = 0 - (\frac{-8}{3} + \frac{4}{2} - 2(-2)) = 0 - (-2.66 + 2 + 4) = 0 - 3.33 = -3.33$. Now ... we get a negative result, this just meaning we considered wrongly that y^2 was above $2 - y$ ... but this gets corrected on the spot, since 3.33 is the right answer (just ignore the "-" and you get the correct answer).

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Again we are faced with the y^2 ... so we need again consider the "unnatural" order of variables. Now ... $2x = y^2 \to x = \frac{y^2}{2}$. Also, $y = x - 4 \to x = y + 4$.

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Find intersection points: $\frac{y^2}{2} = y+4 \to y^2 = 2y+8 \to y^2-2y-8=0 \to (y+2)(y-4)=0 \to y=-2$ and $y=4$. Now we have no restrictions whatsoever ... so we'll just compute the integral using the values we found as limit points: $\int_{-2}^{4}(\frac{y^2}{2}-(y+4))\,dy = \int_{-2}^{4}(\frac{y^2}{2}-y-4)\,dy = \frac{y^3}{6}-\frac{y^2}{2}-4y|_{-2}^{4} = (\frac{64}{6}-\frac{16}{2}-16)-(\frac{-8}{3}-\frac{4}{2}-4(-2)) = (10.66-8-16)-(-1.33-2+8) = -13.33-4.66 = -18$. Again we get a negative answer, but again we correct this by taking 18 to be our answer, droping the "-".

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Now everything is regular! (a) intersection points: $x^2-1 = 2x+2 \to x^2-2x-3=0 \to (x-3)(x+1)=0 \to x=-1$ and $x=3$. Compute the integral: $\int_{-1}^{3}(x^2-2x-3)\,dx = \frac{x^3}{3}-2\frac{x^2}{2}-3x|_{-1}^{3} = (\frac{27}{3}-9-9)-(\frac{-1}{3}-1+3) = -9-(1.66) = -10.66$. Answer is? 10.66. (b) Notice that in the first intersection point, -1, both functions become 0; from that point on the line will shot up, so it's above the x-axis, but the parabola will go below ... it will become 0 again in $x=1$ (solve the equation $x^2-1=0$!!), and from there it will be positive only. So ... what part lies above the x-axis? the parabola has no influence between -1 and 1, so the area will be just the one below the line: $\int_{-1}^{1}(2x+2)\,dx = x^2+2x|_{-1}^{1} = (1+2)-(1-2) = 3-(-1) = 4$. But from 1 on, both function lie above the x-axis, so let's compute integral as in (a): $\int_{1}^{3}(x^2-2x-3)\,dx = \frac{x^3}{3}-x^2-3x|_{1}^{3} = (\frac{27}{3}-9-9)-(\frac{1}{3}-1-3) = -9-(-3.66) = -9+3.66 = -5.33$. So the second area is 5.33. Together, these two areas amount to 4+5.33=9.33, which is $\frac{9.33}{10.66} * 100 = 87.5$ percent.

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