MIDTERM I, FORM A

MATH 132 WI00

I. Compute the following limits (if the limit is $+\infty$ or $-\infty$ or DNE, state whether it is $+\infty$ or $-\infty$ or DNE; leave the answer in fractions)

(a) $\lim_{t\to 2} \sqrt[3]{t^2+1}$ (6 points)

Answer: $\sqrt[3]{5}$ **- plug in 2**

(b) $\lim_{x\to 2} \frac{4-x^2}{x^2-5x+6}$ (6 points)

Answer: $\frac{-4}{-1} = 4$ **since** $\frac{4-x^2}{x^2-5x+6} = \frac{-(x-2)(x+2)}{(x-2)(x-3)} = \frac{-(x+2)}{(x-3)}$

(c) $\lim_{x\to\infty} \frac{x^{1997}+9}{1997x^{1998}+2000}$ (6 points)

Answer: 0 **- it's** $\lim_{x\to\infty} \frac{x^{1997}}{1997x^{1998}} = \lim_{x\to\infty} \frac{1}{1997x}$

(d) $\lim_{x\to-\infty} \frac{x}{1-7x}$ (6 points)

Answer: $-\frac{1}{7}$ **- same as previous, only** $\lim_{x\to-\infty} \frac{x}{-7x} \dots$

(e) Let $f(x) = 5x - 8$, find $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ (6 points)

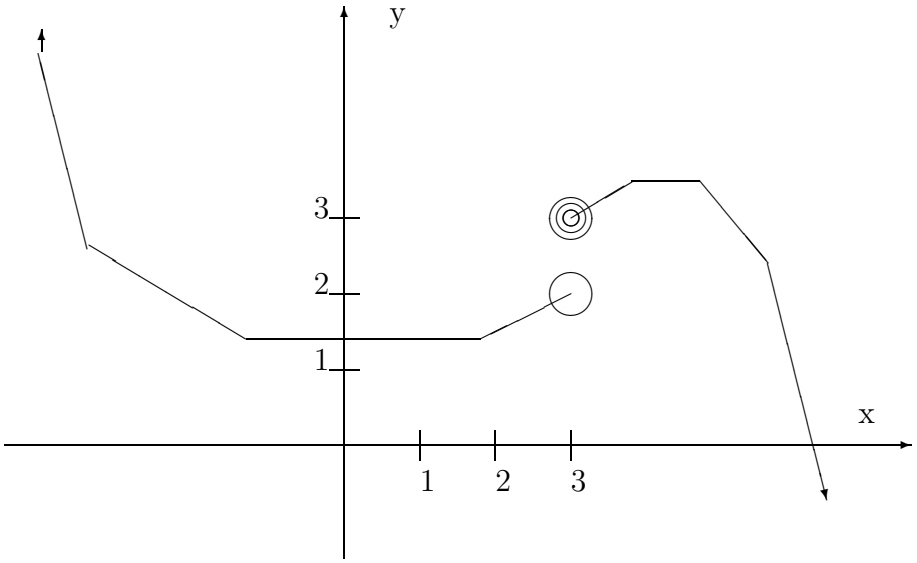
Answer: 5

(f) $\lim_{h\to 0} \frac{\frac{4}{3+h} - \frac{4}{3-h}}{h}$ (6 points)

Answer: $-\frac{8}{9}$

Date: 01/21/2001.

(g) Given the graph of a function $G(x)$ (below)



find:

(i) $\lim_{x \to 3^+} G(x)$ (2 points)

Answer: 3

(ii) $\lim_{x \to 3^-} G(x)$ (2 points)

Answer: 2

(iii) $G(3)$ (2 points)

Answer: 3

(iv) $\lim_{x \to \infty} G(x)$ (2 points)

Answer: ∞

(v) $\lim_{x \to -\infty} G(x)$ (2 points)

Answer: $-\infty$

(h) Let $f(x) = \begin{cases} \frac{7}{x+1}, & \text{if } x < 1 \\ \frac{7}{4}, & \text{if } x = 1 \\ \frac{14}{x+5}, & \text{if } x > 1 \end{cases}$. Find:

(i) $\lim_{x \to 1^+} f(x)$ (2 points)

Answer: $\frac{14}{6}$ **(use last formula)**

(ii) $\lim_{x \to 1^-} f(x)$ (2 points)

Answer: $\frac{7}{2}$ **(use first formula)**

(iii) $f(1)$ (2 points)

Answer: $\frac{7}{4}$ **(use the middle definition)**

II. Find the derivatives of the following functions (do not simplify)

(a) $f(s) = (2s+1)(1-s+3s^2)$ (6 points)

Answer: $2(1-s+3s^2)+(2s+1)(-1+6s)$ **-product rule**

(b) $f(t) = \dfrac{t^3+2}{2t^5+t+4}$ (6 points)

Answer: $\frac{(3t^2)(2t^5+t+4)-(t^3+2)(10t^4+1)}{(2t^5+t+4)^2}$ **- quotient rule**

(c) $y = [(3x-7)^5][(x^2+5)]$ (6 points)

Answer: $[5(3x-7)^4(3)](x^2+5)+[(3x-7)^5](2x)$ **- product rule combined with chain rule**

III. Solve the inequality

$$\frac{(7-x)(5+x)}{(x-3)} \leq 0$$ (10 points)

Answer: $[-5,3) \cup [7,\infty)$

IV. Find the equation of the tangent line to the graph of

$$y = 8x^3 + 7x - 6$$

at the point $(1, 9)$ (8 points)

Answer: slope is given by derivative ($24x^2+7$) in 1 ($24\cdot 1^2+7 = 31$), so it's 31. Equation is given by: $y - 9 = 31(x - 1)$

V. Let $p = 200 - 2q - q^2$ be the demand function for a manufacturers product.

(a) Find the rate of change of price p per unit with respect to quantity q (6 points)

Answer: read the problem carefully - we are asked to find RATE OF CHANGE OF p WITH RESPECT TO q, hence it's DERIVATIVE of p with respect to q - result is $-2 - 2q$

(b) How fast is the price changing with respect q when $q = 5$ (6 points)

Answer: plug in 5 - result is $-2 - 2 \cdot 5 = -2 - 10 = -12$