MIDTERM I, FORM A

MATH 132 WI01

I. Compute the following limits (if the limit is $+\infty$ or $-\infty$ or DNE, state whether it is $+\infty$ or $-\infty$ or DNE; leave the answer in fractions)

$$\text{(i)} \qquad \lim_{x\to 0^-} \frac{3-x}{x} \qquad \text{(6 points)}$$

Proof. Plug in 0, gives you $\frac{3}{0}$ - must be an infinity; check the sign: $3-0>0$ so positive; $x<0$, since $x\to 0^-$, so the fraction is negative. Hence **the answer is** $-\infty$. □

$$\text{(ii)} \qquad \lim_{x\to\infty} \frac{15x^7-9x^2-11}{x^8-5} \qquad \text{(6 points)}$$

Proof. It's a limit to ∞, so we need to ignore everything but the highest power of x in the numerator and the highest power of x in the denominator. This gives:

$$\lim_{x\to\infty}\frac{15x^7}{x^8} = \lim_{x\to\infty}\frac{15}{x} = 0$$

□

$$\text{(iii)} \qquad \lim_{x\to 1} \frac{x^2-2x+1}{x^2+3x-4} \qquad \text{(6 points)}$$

Proof. plug in 1, and get: $\frac{0}{0}$! so we need to simplify first:

$$\frac{x^2-2x+1}{x^2+3x-4} = \frac{(x-1)(x-1)}{(x-1)(x+4)} = \frac{x-1}{x+4}$$

so the limit becomes

$$\lim_{x\to 1}\frac{x^2-2x+1}{x^2+3x-4} = \lim_{x\to 1}\frac{x-1}{x+4} = 0$$

□

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(iv) Let $f(x) = x^2 - 11x - 5$, find $\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ (6 points)

Proof. Let's first compute $f(x+h) = (x+h)^2 - 11(x+h) - 5 = x^2 + 2xh + h^2 - 11h - 11x - 5$; so then

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} =$$

$$= \lim_{h\to 0} \frac{(x^2 + 2xh + h^2 - 11h - 11x - 5) - (x^2 - 11x - 5)}{h} =$$

$$= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - 11h - 11x - 5 - x^2 + 11x + 5)}{h} =$$

$$= \lim_{h\to 0} \frac{2xh + h^2 - 11h}{h} =$$

$$= \lim_{h\to 0} \frac{h(2x + h - 11)}{h} = \lim_{h\to 0}(2x + h - 11) = 2x - 11$$

□

(v) $\lim_{x\to\infty} \frac{7 - x - 11x^2}{5x^2 + 3}$ (6 points)

Proof. Again, a limit to ∞; consider only highest powers of x in both numerator and denominator ...

$$\lim_{x\to\infty} \frac{7 - x - 11x^2}{5x^2 + 3} = \lim_{x\to\infty} \frac{-11x^2}{5x^2} =$$

$$= \lim_{x\to\infty} \frac{-11}{5} = -\frac{11}{5}$$

□

(vi) $\lim_{t\to 0} \frac{\frac{5}{7+t} - \frac{5}{7-t}}{t}$ (6 points)

Proof.

$$\frac{5}{7+t} - \frac{5}{7-t} = \frac{5(7-t) - 5(7+t)}{(7+t)(7-t)} = \frac{-10t}{49-t^2}$$

so the limit becomes

$$\lim_{t\to 0} \frac{\frac{5}{7+t} - \frac{5}{7-t}}{t} = \lim_{t\to 0} \frac{\frac{-10t}{49-t^2}}{t} =$$

$$= \lim_{t\to 0} \frac{-10t}{t(49-t^2)} = \lim_{t\to 0} \frac{-10}{(49-t^2)} =$$

$$= \frac{-10}{49}$$

□

(vii) Given the graph of a function $f(x)$ (below)

(your graph here ... check your paper)
find:

(a) $\lim_{x\to 1^+} f(x)$ (2 points)

Answer: -2

(b) $\lim_{x\to 1^-} f(x)$ (2 points)

Answer: -1

(c) $\lim_{\to 1} f(x)$ (2 points)

Answer: DNE **- no match between the above results**

(d) $\lim_{x\to 5} f(x)$ (2 points)

Answer: 1

(e) $\lim_{x\to\infty} f(x)$ (2 points)

Answer: ∞

II Let $f(x) = \begin{cases} \frac{9}{x+5}, & \text{if } x < 0 \\ \frac{9}{5}, & \text{if } x = 0 \\ \frac{18}{x+10}, & \text{if } x > 0 \end{cases}$. Find:

(a) $\lim_{x \to 0^+} f(x)$ (3 points)

Answer: $\frac{18}{10} = \frac{9}{5}$ **(use third formula)**

(b) $\lim_{x \to 0^-} f(x)$ (3 points)

Answer: $\frac{9}{5}$ **(use first formula)**

(c) $f(2)$ (3 points)

Answer: $\frac{18}{2+10} = \frac{18}{12}$ **since** $2 > 0$**, hence we use the third formula**

II. Find the derivatives of the following functions (do not simplify)

(a) $f(x) = (9 + 11x^5)(9 - 8x^3 + 7x^5)$ (6 points)

Answer: $11 \cdot 5x^4(9 - 8x^3 + 7x^5) + (9 + 11x^5)(-8 \cdot 3x^2 + 7 \cdot 5x^4)$ **-product rule**

(b) $f(s) = \dfrac{s^7 - 5s + 8}{s^4 + 9s - 2}$ (6 points)

Answer: $\frac{(7s^6-5)(s^4+9s-2)-(s^7+9s-2)(4s^3+9)}{(s^4+9s-2)^2}$ **- quotient rule**

(c) $y = \dfrac{70}{\sqrt{9x+7}} - 84$ (6 points)

Answer: $y' = (70 \cdot (9x+7)^{-\frac{1}{2}})' - 84' = 70(-\frac{1}{2})(9x+7)^{-\frac{3}{2}} \cdot 9$ **- power rule combined with chain rule**

(d) $g(x) = (3x+2)^8 - 9x^{\frac{3}{5}} + 18$ (6 points)

Answer: $8(3x+2)^7 \cdot 3 - 9\frac{3}{5}x^{-\frac{2}{5}}$

III. Solve the inequality

$$\frac{(5-x)(6+x)}{(x-7)} \leq 0 \qquad \text{(10 points)}$$

Proof. Draw the sign table, and point out the numbers at which either the numerator or the denominator become zero, namely -6, 5 and 7. We get:

x		-6		5		7	
$fraction$	$+$	0	$-$	0	$+$	DNE	$-$

(plug in -7, 0, 6 and 8 in f to find the signs above)

The answer is, hence, $[-6,5] \cup (7,\infty)$ (do not include 7, since the fraction is not defined in 7; but do include the other endpoints, since it's $\leq$) ☐

IV. Find an equation of the tangent line to the graph of

$$y = 9x^3 + 8x - 5$$

at the point $(1,12)$ (6 points)

Answer: slope is given by derivative $(27x^2+8)$ **in 1** $(27\cdot 1^2+8 = 35)$**, so it's 35. Equation is given by:** $y - 12 = 35(x-1)$

V. If a manufacturer's *Revenue function* is given by

$$R = 4q^2 + 5q$$

(6 points)

(a) Find the marginal revenue function

Answer: it's DERIVATIVE of R **with respect to** q **- result is** $4 \cdot 2q + 5 = 8q + 5$

(b) Find the marginal revenue when $q = 10$ units are produced

Answer: plug 10 in the above result - value is $8 \cdot 10 + 5 = 85$