MIDTERM III

MATH 132 WINTER 2000

I. Evaluate the following integrals

a) $$\int x(7x^2+2)^4\,dx \qquad (10p)$$

use substitution: $u = 7x^2+2) \Rightarrow du = 7\cdot 2x\,dx = 14x\,dx \Rightarrow \frac{1}{14}\,du = x\,dx$.
The integral then becomes:

$$\int u^4 \frac{1}{14}\,du = \frac{1}{14}\int u^4\,du = \frac{1}{14}\frac{u^5}{5} + C = \frac{(7x^2+2)^5}{70} + C$$

b) $$\int \frac{x^2+6x+3}{x}\,dx \qquad (10p)$$

divide each term of the numerator by x, and get:

$$\int (x+6+3\frac{1}{x})\,dx = \int x\,dx + \int 6\,dx + \int 3\frac{1}{x}\,dx = \frac{x^2}{2} + 6x + 3\ln(x) + C$$

c) $$\int_0^6 100+6x^2\,dx \qquad (10p)$$

$$\int 100+6x^2\,dx = 100x + 6\frac{x^3}{3} = 100x + 2x^3$$

so

$$\int_0^6 100+6x^2\,dx = (100x+2x^3)|_0^6 = (100\cdot 6 + 2\cdot 6^3) - (100\cdot 0 + 2\cdot 0^3) =$$

$$= (600+432) - 0 = 1032$$

d) $$\int \frac{e^x}{e^x+5}\,dx \qquad (10p)$$

Date: 03/03/2000.

Use substitution again, namely $u = e^x + 5 \Rightarrow du = e^x\,dx$ so our integral becomes

$$\int \frac{1}{u}\,du = \ln(u) + C = \ln(e^x + 5) + C$$

(at this point have the integral tables - in Appendix C of your book - ready)

e) use provided integral tables to find the following integral. State the number of formula that you use

$$\int \frac{x\,dx}{\sqrt{5+6x}} \qquad\qquad 15\text{p}$$

if you have the samples' package, the formula that matches our function is **8.**; if you only have your textbook, the formula in Appendix C that matches the function is **15.**

based on this formula we get, noticing that $a = 5$ and $b = 6$:

$$\int \frac{x}{\sqrt{5+6x}}\,dx = \frac{2(6x - 2\cdot 5)\sqrt{5+6x}}{3\cdot 6^2} + C = \frac{2(6x-10)\sqrt{5+6x}}{108} + C$$

II. Find the area of the region bounded by: the graph of the curve $y = x^2 - 2x$, the lines $x = -2$, $x = 1$ and the x-axis (15p)

Draw the graph! (in the exam, the graph, most likely, will be provided to you ...)

We have to integrate from $x = -2$ up to $x = 1$. Let's check whether y becomes zero between -2 and 1: $y = 0 \Rightarrow x^2 - 2x = 0 \Rightarrow x(x-2) = 0 \Rightarrow x = 0$ or $x = 2$. Between -2 and 1 there's only one of them, namely 0. So let's compute the integral from -2 to 0

$$\int_{-2}^{0} x^2 - 2x\,dx = \frac{x^3}{3} - x^2|_{-2}^{0} = (\frac{0^3}{3} - 0^2) - (\frac{(-2)^3}{3} - (-2)^2) =$$

$$= 0 - \frac{-8}{3} + 4 = 2.66 + 4 = 6.66$$

and the integral from 0 to 1

$$\int_{0}^{1} x^2 - 2x\,dx = \frac{x^3}{3} - x^2|_0^1 = (\frac{1^3}{3} - 1^2) - (\frac{0^3}{3} - 0^2) = \frac{1}{3} - 1 = 0.33 - 1 = -0.66$$

Notice that the second integral is **negative!** this because the graph of y is below the x-axis; in order to compute the total area, on the other hand, we just need to drop the second integral's negative sign, and add the two, positive now, numbers together:

$$Total\_area = 6.66 + 0.66 = 7.33$$

III. Suppose the demand equation for a product is given by

$$p = 400 - 2q$$

and its supply function is given by

$$p = q + 100$$

(a) find the equilibrium point
(b) find the consumer's surplus
(c) find the producer's surplus (15p)

it's the regular setup - with p in terms of q for both equations. Draw the graph(s)!

We need the intersection point - set the two equations equal to one another:

$$400 - 2q = q + 100 \Rightarrow 400 = q + 100 + 2q = 3q + 100 \Rightarrow 400 - 100 = 3q \Rightarrow$$
$$\Rightarrow 300 = 3q \Rightarrow q = 100$$

and based on this, $p = q + 100 = 100 + 100 = 200$. Hence the equilibrium is $(p, q) = (200, 100)$.

To find the Consumer's Surplus, look at the graph - it's the upper-most area, bounded by the decreasing $p = 400 - 2q$ function and by $p = 200$ (the equilibrium). The integral goes from $q = 0$ up to $q = 100$ (again, the equilibrium). Hence we have:

$$CS = \int_0^1 00[(400 - 2q) - 200]\, dq = \int_0^1 00200 - 2q\, dq = (200q - q^2)|_0^1 00 =$$
$$= (200 \cdot 100 - 100^2) - (200 \cdot 0 - 0^2) = (20000 - 10000) - 0 = 10000$$

For the Producer's Surplus, we look at the area below the Consumer's Surplus' area - it's bounded by $p = 200$ (the equilibrium) and the increasing $p = q + 100$ function; also, this integral goes, again, from $q = 0$ to $q = 100$ (the equilibrium, once again). We get:

$$PS = \int_0^1 00[200-(q+100)]\, dq = \int_0^1 00(200-q-100)\, dq = \int_0^1 00100-q\, dq =$$
$$= (100q - \frac{q^2}{2})|_0^1 00 = (100 \cdot 100 - \frac{100^2}{2}) - (100 \cdot 0 - \frac{0^2}{2}) =$$
$$= (10000 - 5000) - 0 = 5000$$

IV. Find the area under the graph of the curve $y = 9 - x^2$ and above the line $y = 5 - 3x$. (15p)

Draw the graph! (again, most likely the exam paper will provide you with the graph). Let's see where the two graphs intersect: $9 - x^2 = 5 - 3x \Rightarrow 0 =$

$5 - 3x - 9 + x^2 = x^2 - 3x - 4 = (x-4)(x+1)$ which gives $x = -1$ os $x = 4$. Hence, we're interested in the region bounded by these two values. We need to compute the integral of the **difference** between the two functions, from -1 to 4. Optionally, we need to see which function sits above (why optionally? if you choose any order, and you get a negative result, this just means you chose the wrong order; but the result you got is still OK, just drop the sign!). As an example, let's choose the obviously wrong order, namely suppose the line is above, so we substract $y = 9 - x^2$ out of $y = 5 - 3x$.

Let's compute:

$$\int_{-1}^{4} [(5-3x)-(9-x^2)]\,dx = \int_{-1}^{4} (5-3x-9+x^2)\,dx = \int_{-1}^{4} (x^2-3x-4)\,dx =$$

$$= (\frac{x^3}{3} - 3\frac{x^2}{2} - 4x)|_{-1}^{4} = (\frac{4^3}{3} - 3\frac{4^2}{2} - 4\cdot 4) - (\frac{(-1)^3}{3} - 3\frac{(-1)^2}{2} - 4\cdot(-1)) =$$

$$= (21.33 - 24 - 16) - (-0.33 + 1.5 + 4) = -18.66 - 5.16 = -23.83$$

Negative result? No problem, it means we just chose the right order ... so the actual result reads:

$$area\_between\_graphs = 23.83$$