MIDTERM III

MATH 132 WINTER 2001

I. Evaluate the following integrals

$$\text{a)} \qquad \int \frac{1}{x-1} + \frac{1}{(x-1)^2}\,dx \qquad \text{(14p)}$$

use substitution: $u = x - 1 \Rightarrow du = dx$. The integral then becomes:

$$\int \frac{1}{u} + \frac{1}{u^2}\,du = \int \frac{1}{u}\,du + \int u^{-2}\,du =$$
$$= \ln|u| + \frac{u^{-1}}{-1} + C = \ln|x-1| + \frac{(x-1)^{-1}}{-1} + C$$

$$\text{b)} \qquad \int \frac{3e^x}{6+5e^x}\,dx \qquad \text{(14p)}$$

substitution: $u = 6 + 5e^x \Rightarrow du = 5e^x\,dx \Rightarrow \frac{1}{5}\,du = e^x\,dx \Rightarrow \frac{3}{5}\,du = 3e^x\,dx$. Hence the integral becomes:

$$\int \frac{\frac{3}{5}}{u}\,du = \frac{3}{5}\int \frac{1}{u}\,du =$$
$$= \frac{3}{5}\ln|u| + C = \frac{3}{5}\ln|6+5e^x| + C$$

$$\text{c)} \qquad \int_0^1 x^2 + 4x + 2\,dx \qquad \text{(14p)}$$

$$\int x^2 + 4x + 2\,dx = \frac{x^3}{3} + 4\frac{x^2}{2} + 2x = \frac{x^3}{3} + 2x^2 + 2x$$

so

$$\int_0^1 x^2 + 4x + 2\,dx = (\frac{x^3}{3} + 2x^2 + 2x)|_0^1 =$$
$$= (\frac{1^3}{3} + 2\cdot 1^2 + 2\cdot 1) - 0 = .33 + 2 + 2 = 4.33$$

Date: 03/03/2001.

II. Find the area of the shaded region bounded by the graph of the curves $y = 5 - x^2$ $y = 4x$ (14p)

We have to find the intersections between the two graphs - so set the equations equal to one another.

$$5 - x^2 = 4x \Rightarrow 0 = x^2 + 4x - 5 = (x-1)(x+5)$$

and hence $x = -5$ and $x = 1$ are the x-values of the intersections. This just means that we will have to integrate from -5 to 1 the difference between the two functions (as an idea, the only reason you should look at the graph given to you should be to check which function sits above and which below ... ignore everything else!). We see that the parabola is above. Let's compute:

$$\int_{-5}^{1} (5 - x^2) - 4x\,dx = \int_{-5}^{1} 5 - 4x - x^2\,dx =$$
$$= 5x - 4\frac{x^2}{2} - \frac{x^3}{3}|_{-5}^{1} = 5x - 2x^2 - \frac{x^3}{3}|_{-5}^{1} =$$
$$= (5 \cdot 1 - 2 \cdot 1^2 - \frac{1^3}{3}) - (5 \cdot (-5) - 2 \cdot (-5)^2 - \frac{(-5)^3}{3}) =$$
$$= 2.66 + 33.33 = 36$$

III. Find the area of the shaded region bounded by the graph of the curve $y = 6 - x - x^2$, the x-axis, and the lines $x = 0$ and $x = 3$.

First we need to see where the curve intersects the x-axis: $6 - x - x^2 = 0 \Rightarrow 0 = x^2 + x - 6 \Rightarrow (x-2)(x+3) = 0 \Rightarrow x = -3$ or $x = 2$.

Let's see which one is between 0 and 3: -3 isn't, but 2 is - this means we have to break our $[0, 3]$ interval in two, $[0, 2]$ and $[2, 3]$ (mind the fact that we needed to compute the intersection, and find the 2; just taking it from the graph is not enough! - think of it like that: if it were 2.01, the graph would still show you the 2 ...). Let's compute integrals:

$$\int_{0}^{2} 6 - x - x^2\,dx = 6x - \frac{x^2}{2} - \frac{x^3}{3}|_0^2 =$$
$$= (6 \cdot 2 - \frac{2^2}{2} - \frac{2^3}{3}) - 0 = 7.33$$
$$\int_{2}^{3} 6 - x - x^2\,dx = 6x - \frac{x^2}{2} - \frac{x^3}{3}|_2^3 =$$
$$= (6 \cdot 3 - \frac{3^2}{2} - \frac{3^3}{3}) - (6 \cdot 2 - \frac{2^2}{2} - \frac{2^3}{3}) =$$
$$= 4.5 - 7.33 = -2.83$$

The second result is negative, but this is because the graph is below the x-axis; just drop the $-$ sign and add the two results:

$$Total\_Area = 7.33 + 2.83 = 10.16$$

IV. Use provided integral tables to find the following integral. State the number of formula that you use.

$$\int \frac{dx}{(5x+2)\sqrt{4+9(5x+2)}}$$

(14 points)

well ... the repeated use of $5x+2$ is a hint to use it as u: $u = 5x+2 \Rightarrow du = 5\,dx \Rightarrow dx = \frac{1}{5}\,du$; our integral becomes:

$$\int \frac{\frac{1}{5}\,du}{u\sqrt{4+9u}}$$

and take out the $\frac{1}{5}$ and use formula #9

$$\int \frac{\frac{1}{5}\,du}{u\sqrt{4+9u}} = \frac{1}{5}\int \frac{du}{u\sqrt{4+9u}} =$$

$$= \frac{1}{5}\cdot\frac{1}{\sqrt{4}}\ln|\frac{\sqrt{4+9u}-\sqrt{4}}{\sqrt{4+9u}+\sqrt{4}}| + C =$$

$$= \frac{1}{5}\cdot\frac{1}{2}\ln|\frac{\sqrt{4+9(5x+2)}-2}{\sqrt{4+9(5x+2+2}}| + C$$

III. Suppose the demand equation for a product is given by

$$p = 100 - 0.05q$$

and its supply function is given by

$$p = 10 + 0.1q$$

(a) find the equilibrium point
(b) find the consumer's surplus (16p)

it's the regular setup - with p in terms of q for both equations. Draw the graph(s)!

We need the intersection point - set the two equations equal to one another:

$$100 - 0.05q = 10 + 0.1q \Rightarrow 100 = 0.05q + 10 + 0.1q = 0.15q + 10 \Rightarrow$$

$$\Rightarrow 100 - 10 = 0.15q \Rightarrow 90 = 0.15q \Rightarrow q = 600$$

and based on this, $p = 100-0.05q = 100-30 = 70$. Hence the equilibrium is $(q,p) = (600, 70)$.

To find the Consumer's Surplus, look at the graph - it's the upper-most area, bounded by the decreasing $p = 100 - 0.05q$ function and by $p = 70$

(the equilibrium). The integral goes from $q = 0$ up to $q = 600$ (again, the equilibrium). Hence we have:

$$\begin{aligned} CS = \int_0^{600} [(100 - 0.05q) - 70]\, dq = \int_0^{600} 30 - 0.05q\, dq = \\ = (30q - \frac{0.05}{2}q^2)|_0^{600} = \\ = (30 \cdot 600 - \frac{0.05}{2}600^2) - (30 \cdot 0 - \frac{0.05}{2}0^2) = \\ = (18000 - \frac{5}{200}36000) - 0 = 18000 - 900 = 17100 \end{aligned}$$