SAMPLE MIDTERM II HINTS AND SOLUTIONS

MATH 132 - WI '00

1.

$f'(x) = 2x * e^{-x} + x^2 * e^{-x} * (-1) = 2xe^{-x} - x^2e^{-x} = xe^{-x}(2-x)$ (always a good idea to factor out common factors, so you get a product of simple factors)

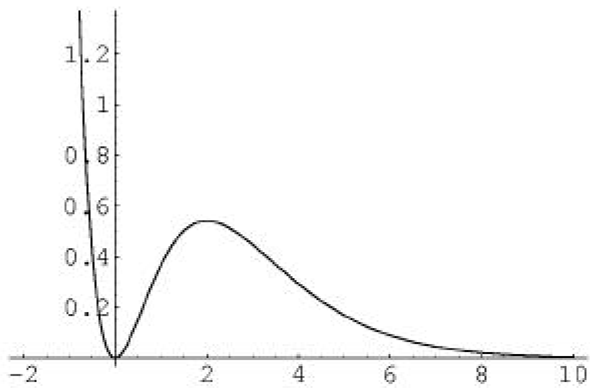
When is $f'(x) = 0$?

$x = 0$, e^{-x} is never 0, $2 - x = 0 \to x = 2$. Let's draw the sign table (for the signs plug -1, 1 and 3)

y': $-\infty$	$(-)$	0	$(+)$	2	$(+)$	∞.
y: $-\infty$	$\searrow$	0	$\nearrow$	2	$\searrow$	∞.

From this we get that 0 is a relative minimum, and 2 is a relative maximum.

Here's the graph:



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2.

As you can see, you don't need to compute the derivatives anymore, since they're already there! you notice that ± 1 are critical numbers (they make $f' = 0$) and $\pm\sqrt{3}$ are values that make $f'' = 0$. Draw the sign tables, for both f' and f''. Afterwards, plot the points you are given! right at the beiginning of the problem, and connect them by curves, following the info you have in the tables. (We'll discuss this problem most likely in class on Tuesday, if there are still questions about it).

Date: 02/11/2000.

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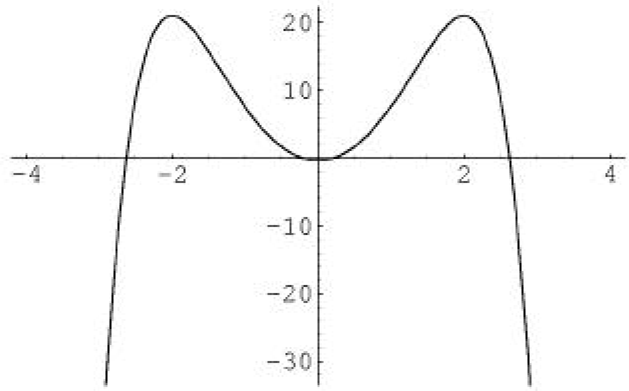
3.

Compute the derivative of y: $y' = 0+8*2x-\frac{1}{6}*6x^5 = 16x-x^5 = x(16-x^4)$. Critical numbers are those x-s that make $y' = 0$: $x = 0$ and $16 - x^4 = 0 \to 16 = x^4 \to$ (square root) $\pm 4 = x^2$; but x^2 is POSITIVE, so only choice is $4 = x^2 \to x = \pm 2$. These are critical numbers; draw the sign-table for y', and this will also point out what kind of relative extrema they are. For points of inflexion we need $y'' = (16x - x^5)' = 16 - 5x^4$. $y'' = 0 \to 16 - 5x^4 = 0 \to 16 = 5x^4 \to x^4 = \frac{16}{5} \to x^2 = \pm\sqrt{\frac{16}{5}} \to x^2 = \sqrt{\frac{16}{5}} = 1.7888 \to x = \sqrt{1.7888} = \pm 1.33$. Draw the sign table for y'' ... and that will tell you what kind of points of inflexion they are (plug -2, 0, 2 in y'', for example). To help you, here's the graph:



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4.

Profit=Revenue-Cost$=100x-\frac{x^2}{2}-(1200+12x+\frac{x^2}{2}) = 100x-\frac{x^2}{2}-1200-12x-\frac{x^2}{2} = -1200+88x-x^2$. To find maximum we need (Profit)'$=88-2x=0 \to 88=2x \to x=44$. To check whether it is MAXIMUM indeed (could be minimum as well!) let's use second derivative test: (Profit)"$=(88-2x)' = -2 \to$ negative for any x $\to$ ($\frown$) $\to$ maximum. **Note: it's IMPORTANT to check whether some critical number is maximum or minimum - I bet there'll be points taken off in the midterm if you don't do that!**.

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5.

Draw the field + fence $\to$ calling length L and width W, the length of fence is: one L and two Ws $\to 5000 = L + 2W \to L = 5000 - 2W$. Area is $L * W = (5000-2W)*W = 5000W-2W^2$. To maximize it, we need (Area)'$=5000-2*2W = 5000-4W = 0 \to 5000 = 4W \to W = \frac{5000}{4} \to W = 1250 \to L = 5000 - 2W = 5000 - 2500 = 2500$.

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6.

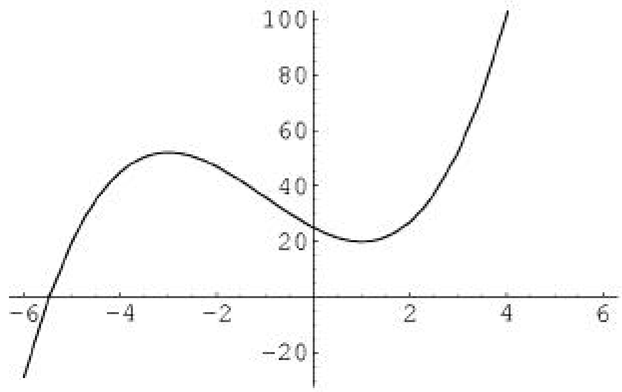
(a) Plug 0 in f.

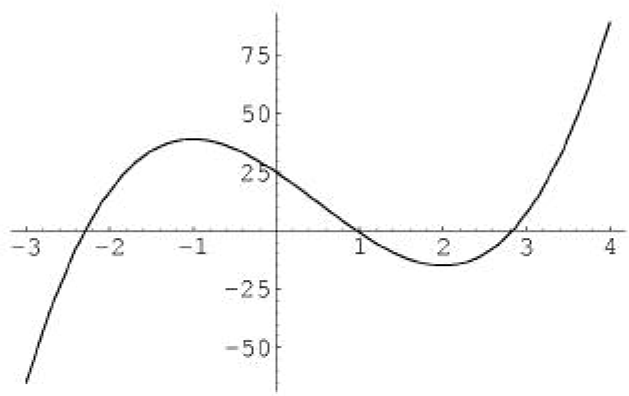
(b),(c)(i) $f' = 3x^2 + 3 * 2x - 9 = 3x^2 + 6x - 9 = 3(x^2 + 2x - 3) = 3(x-1)(x+3)$; (ii) $f' = 4*3x^2 - 6*2x - 24 = 12x^2 - 12x - 24 = 12(x^2 - x - 2) = 12(x-2)(x+1)$.Critical numbers ... sign-table ... have fun!

(d),(e) (i) $f'' = 3(2 * x + 2) = 3 * 2(x + 1) = 6(x + 1)$;

(ii) $f'' = 12(2x - 1)$. Make them equal to 0 ... sign table ...

(f)Here are the graphs:





7.

(a) First, $f' = 3x^2 - 3 * 2x - 9 = 3x^2 - 6x - 9 = 3(x^2 - 2x - 3) = 3(x-3)(x+1) \rightarrow x = 3, x = -1$. To find out what kind of relative extrema they are, let's compute $f'' = 3(2x-2) = 3*2(x-1) = 6(x-1)$. Plug 3 in: $f''(3) = 6*(3-1) = 6*2 = 12 > 0 \rightarrow$ *positive* $\rightarrow (\smile) \rightarrow$ minimum. Plug -1 in :$f''(-1) = 6(-1-1) = 6*(-2) = -12 < 0 \rightarrow$ *negative* $\rightarrow (\frown) \rightarrow$ maximum.

(b) The only critical number in (-3,3) is -1. Plug, hence, -3, -1 and 3 in f: we get $f(-3) = (-3)^3 - 3(-3)^2 - 9(-3) + 16 = -11$, $f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 16 = 21$, $f(3) = 3^3 - 3 * 3^2 - 9 * 3 + 16 = -11$. Hence ...

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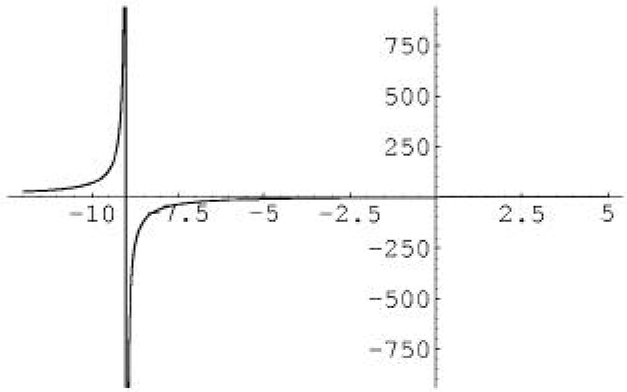
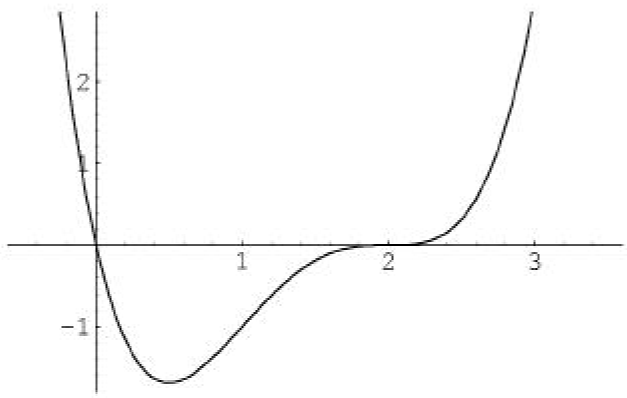
8.

Let me just give you the derivatives and the graphs ... by now you know what to do! (i) $f' = \frac{7*(x+9) - 7x*1}{(x+9)^2} = \frac{-63}{(x+9)^2}$ (always negative!! ... hence f always decreasing)



(ii) $f' = 1*(x-2)^3 + x*3(x-2)^2*1 = (x-2)^3 + 3x(x-2)^2 = (x-2)^2*(x-2+3x) = (x-2)^2*(4x-2) = 2(2x-1)(x-2)^2$. (find zeroes, notice that $(x-2)^2$ is ALWAYS positive, so you can ignore it ...)



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9.

(a) $\int x^5 + x - 2 = \int x^5 + \int x - \int 2 = \frac{x^6}{6} + \frac{x^2}{2} - 2 * \int x^0 = \frac{x^6}{6} + \frac{x^2}{2} - 2 * \frac{x^1}{1} + C$

(b) $\int \frac{1}{7x^2} = \frac{1}{7} \int x^{-2} = \frac{1}{7} \frac{x^{-1}}{-1} + C$

(c) Let's first simplify expresion in integral:

$\frac{6x^4}{(3x^5)^3} = \frac{6x^4}{3^3*(x^5)^3} = \frac{6x^4}{27x^{5*3}} = \frac{6x^4}{27x^15} = \frac{6}{27}\frac{x^4}{x^15} = \frac{6}{26}x^{4-15} = \frac{6}{27}x^{-11}$.

Now, the integral: $\int \frac{6x^4}{3x^5)^3} = \int \frac{6}{27}x^{-11} = \frac{6}{27}\frac{x^{-11+1}}{-11+1} = \frac{6}{27}\frac{x^{-10}}{-10} = -\frac{6}{270x^10} + C$.

(d) $\int \frac{x^4-3x^2+4x}{x} = \int x^3 - 3x + 4 = \frac{x^4}{4} - 3\frac{x^2}{2} + 4\frac{x^1}{1} + C$

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FOR 10, 11 ... same as before; plus, you have to plug the number they give you (1, 2 and 0, respectively) in the RESULTING function, and solve for C.

As you notice, 12 is same as 7 :).

For 13 don't forget: $(ln(x))' = \frac{1}{x}, (e^x)' = e^x$ and, more than that, when you differentiate $(e^{f(x)})' = e^{f(x)} * f'(x)$, that is, WRITE DOWN A COPY OF THE (EXPONENTIAL) FUNCTION, and then differentiate the function you have in the power. For $(ln(f(x)))' = \frac{1}{f(x)} * f'(x)$. And use **product rule, quotien rule** extensively.

Short hint for 13.(d): $ln(x * sqrt3x - 2) = ln(x) + ln(sqrt3x - 2) = ln(x) + ln((3x - 2)^{\frac{1}{2}}) = ln(x) + \frac{1}{2}ln(3x - 2)$ and differentiate this (VERY SIMPLIFIED) version.

See you Tuesday!!