SOLUTIONS SHEET FOR SAMPLE PROBLEMS MIDTERM I

MATH 132 WI01

I. Compute the following limits (if the limit is $+\infty$ or $-\infty$ or DNE, state whether it is $+\infty$ or $-\infty$ or DNE; leave the answer in fractions)

(a) $$\lim_{x\to 2} \frac{4-x^2}{x^2-x-2}$$

Answer: $-\frac{4}{3}$

(b) $$\lim_{x\to \infty} \frac{x^2+6x+9}{(7x-5)^2}$$

Answer: $\frac{1}{49}$

(c) $$\lim_{x\to 3^+} \sqrt{x-3}$$

Answer: 0

(d) $$\lim_{x\to -\infty} \frac{5x-7}{x-7}$$

Answer: 5

(e) $$\lim_{t\to 2} \frac{7t-4}{t-2}$$

Answer: DNE

(f) $$\lim_{h\to 0} \frac{\frac{5}{4+h}-\frac{5}{4}}{h}$$

Answer: $-\frac{5}{16}$

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(h) $\lim_{x \to -5} \dfrac{x^2 - 25}{x - 5}$

Answer: 0

(i) If $f(x) = 11x^2 + 7$, find $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

Answer: $22x$

(j) $\lim_{x \to -\infty} \dfrac{4 - x^2}{1 - x}$

Answer: $-\infty$

(k) $\lim_{x \to \infty} \dfrac{5 - x^2}{x^4 - 8x^2 + 2}$

Answer: 0

(l) $\lim_{x \to 0^-} \dfrac{x^2 + 4}{x}$

Answer: $-\infty$

(m) $\lim_{x \to 1^-} \dfrac{7}{1 - x}$

Answer: ∞

(n) $\lim_{x \to 1} \dfrac{7}{1 - x}$

Answer: DNE

(o) $\lim_{x \to 0} \dfrac{7}{x}$

Answer: DNE

(p) $\lim_{x\to 0} \frac{7}{x^2}$

Answer: ∞

II. Let

$$f(x) = \begin{cases} 5, & \text{if } x > 4 \\ x, & \text{if } x \leq 4 \end{cases}$$

(a) Find $\lim_{x\to 4^+} f(x)$

Answer: 5

(b) Find $\lim_{x\to 4^-} f(x)$

Answer: 4

(c) Find $\lim_{x\to 4} f(x)$

Answer: DNE

(d) Find $\lim_{x\to \infty} f(x)$

Answer: 5

(e) Find $\lim_{x\to -\infty} f(x)$

Answer: $-\infty$

III. Find the derivatives of the following functions (do not simplify)

(a) $f(x) = (2 + 15x - 7x^2)(x^9 - 8x + 4)$

Answer: $(15 - 14x)(x^9 - 8x + 4) + (2 + 15x - 7x^2)(9x^8 - 8)$

(b) $f(t) = \dfrac{t^3 + 7t + 5}{t^5 - 9t^2 + 3}$

Answer: $\frac{(3t^2+7)(t^5-9t^2+3)-(t^3+7t+5)(5t^4-18t)}{(t^5-9t^2+3)^2}$

(c) $y = (8x + 5)^{\frac{4}{7}} + e^{\frac{4}{7}}$

Answer: $\frac{4}{7}(8x + 5)^{-\frac{3}{7}}(8)$

(d) $p = \dfrac{5}{q}$

Answer: $-\frac{5}{2}q^{-\frac{3}{2}}$

(e) $y = \dfrac{(3x + 1)^5 - 5x}{7}$

Answer: $\frac{1}{7}[5(3x + 1)^4(3) - 5]$

(f) $s = (5t^3 + 1)^7(3t - 4)^6$

Answer: $[7(5t^3 + 1)^6(15t^2)](3t - 4)^6 + (5t^3 + 1)^7[6(3t - 4)^5(3)]$

IV. Find the equation of the tangent line to the graph of $y = x^3 - 7x + 6$ at the point $(1, 0)$

Answer: $y = -4x + 4$

V. Find the rate of change of $f(x) = 9x^6 - 8x^3 + 11$ with respect to x and evaluate it when $x = 1$

Answer: $54x^5 - 24x^2$**;** 30

VI. If a manufacturer's average cost function is given by

$$\overline{C} = .005q^2 - .5q + 70 + \frac{300}{q},$$

find the marginal cost function. What is the marginal cost when 50 units are produced?

Answer: $.015q^2 - q + 70$**;** 57.5

VII. Use *Chain Rule* to find $\frac{dy}{dx}$ where $y = 3u^8 - 4$ and $u = 7x^3 + 3x - 1$

Answer: $24u^7(21x^2 + 3) = 24(7x^3 + 3x - 1)^7(21x^2 + 3)$

VIII. Use *Chain Rule* to find $\frac{dy}{dx}$ where $y = 11u^5 - 7u + 11$ and $u = 9x^6 - 8x^3 + 11$

Answer: $(55u^4 - 7)(54x^5 - 24x^2) = [55(9x^6 - 8x^3 + 11)^4 - 7](54x^5 - 24x^2)$

IX. Solve the inequality

(a) $$\frac{(2-x)(x-5)}{11-x} > 0$$

Answer: $(2, 5) \cup (11, \infty)$

(b) $$\frac{(7-x)(x-9)}{8-x} \leq 0$$

Answer: $(-\infty, 7] \cup (8, 9]$