

QUIZ # 1

MATH 132 WI01

Name (1p):

Problem: Let f be the following function:

$$f(x) = \begin{cases} \frac{x+x^2}{-x-x^2} & \text{if } x > 0 \\ \frac{-x-x^2}{x+x^2} & \text{if } x < 0 \end{cases}$$

Compute the following limits:

a) (6p)

$$\lim_{x \rightarrow 0} f(x)$$

Answer: because of multiple definitions, we need to compute partial limits, left and right:

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{-x - x^2}{x + x^2} = \lim_{x \rightarrow 0^-} \frac{-x(1+x)}{x(1+x)} = \lim_{x \rightarrow 0^-} \frac{-(1+x)}{(1+x)} = -1$$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \frac{x + x^2}{-x - x^2} = \lim_{x \rightarrow 0^+} \frac{x(1+x)}{-x(1+x)} = \lim_{x \rightarrow 0^+} \frac{(1+x)}{-(1+x)} = -1$$

Since the two limits are the same, we can conclude that

$$\lim_{x \rightarrow 0} f(x) = -1$$

b) (3p)

$$\lim_{x \rightarrow \infty} f(x)$$

Answer: we take first definition, since ∞ means definitely bigger than 0.

$$\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \frac{x + x^2}{-x - x^2} = \lim_{x \rightarrow \infty} \frac{x^2}{-x^2} = \lim_{x \rightarrow \infty} -1 = -1$$