QUIZ # 4

MATH 132 WI01

Name (1 point): ....

Problem: Let $f(x) = x^e e^x$.

(a) Find intervals of increase/decrease of f (3 points)

(b) Find relative extrema (3 points)

(c) Find y-intercept, and sketch the graph (you can use straight lines to connect points!) (3 points)

NOTICE: This problem turned out to be not exactly what I wanted :) it has a solution, though, so you might want to read it - it's quite instructive, but I doubt very much that you will see anything similar in your exams (so don't worry about it!)

Proof: (a) Before doing anything else, let's see what values of x can be plugged into f (that is, find the **domain** of f). Any positive x will do; same $x = 0$ (since $0^e \cdot e^0 = 0 \cdot 1 = 0$). But negative values won't be good! (think of it like that: if the power is odd, then negative numbers raised to it will stay negative; but if the power is even, then the result is positive; example: $(-1)^2 = 1$; $(-1)^3 = -1$; but the power we have is $e = 2.71821828...$, so what can you choose? positive or negative? **neither, the result Does Not Exist**! in fact, it's a complex number). So we only have to consider $x \geq 0$.

Compute now the derivative of f:

$$f'(x) = ex^{e-1}e^x + x^e e^x = (ex^{e-1} + x^e)e^x = (e \cdot x^{e-1} + x \cdot x^{e-1})e^x$$

$$f'(x) = (e + x)x^{e-1}e^x$$

Since $x \geq 0$ $f'(x) \geq 0$ for all x (the paranthesys is actually bigger than e; x^{e-1} is also positive, since a positive number raised to any power remains positive; e^x is **always** positive) it means that f is increasing on its domain $(x \geq 0)$

(b) we have a relative minimum in 0 ($f'(x) = 0 \rightarrow x = 0$); it's a minimum since the function **increases** from it.

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(c)y-intercept is obtained by plugging in 0 in f:

$$f(0) = 0^e e^0 = 0 \cdot 1 = 0,$$

so $(0, 0)$ is the y-intercept.

Since we don't really care for the shape of the function, let's just sketch it with lines (plot the points obtained until now, and connect them). We only have one point, that is the starting point, $(0, 0)$, but we know that afterwards the function increases, so you can just draw a line with positive slope. I chose to also approximate $(1, f(1)) = (1, e) = (1, 2.71)$, and connect these two points, and draw another line from that second point on. Here's the result.