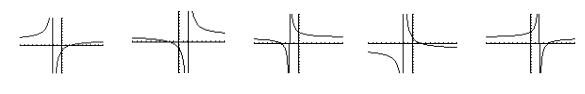
Final Exam

## Part I

- 1. Let g(x) = 2x 3 and  $f(x) = x^2 + 4$ .
  - a) Find the domain of  $\frac{f(x)}{g(x)}$ .
  - b) Find f(4) g(4).
  - c) Find a formula for  $(f \circ g)(x)$ .
  - d) Find all x for which f(x) + g(x) = 0
- **2.** Find all real numbers x which satisfy the inequality:  $x^2 + 3x > 10$ .
- **3.** Factor the polynomial  $f(x) = x^3 13x + 12$  into a product of three **linear** factors. (Hint: 3 is a zero.) Work must be shown. Calculator answers will not receive full credit.
- **4.** Consider the function  $f(x) = \frac{4(x-5)(x+3)}{(x+2)(x-3)}$ . You must do parts a), b), and c) algebraically.
  - a) Find the vertical asymptotes.
  - b) Find the horizontal asymptotes, if any.
  - c) Find the x-intercepts and y-intercepts for the graph.
  - d) Sketch the graph. Be sure to include and label all of the information found above.
- **5.** Under each graph picture write the letter of the appropriate description. There is only one correct description for each picture.
  - A. Horizontal asymptote at f(x) = 2; Vertical asymptote at x = 2; f(x) is decreasing
  - B. Horizontal asymptote at f(x) = 2; Vertical asymptote at x = -2; f(2) = 0
  - C. Horizontal asymptote at f(x) = -2; Vertical asymptote at x = 2; When x > 2, y > 0.
  - D. Horizontal asymptote at f(x) = -2; Vertical asymptote at x = -2; When x < -2, y < 0.
  - E. Horizontal asymptote at f(x) = 2; Vertical asymptote at x = 2; f(x) is increasing
  - F. Horizontal asymptote at f(x) = 2; Vertical asymptote at x = -2; f(0) = 4



## Part II

- **6.** Solve the equation  $1 \log(x + 3) = \log(x)$  by finding the exact solution(s) using **algebraic methods**. (Calculator solutions receive no credit.)
- 7. a) Sketch the graph  $f(x) = 2 \ln x + 2$ . Be sure to label all the intercepts and asymptotes if any.
  - b) Find the formula for the inverse function  $f^{-1}$ .
- **8.** A certain angle  $\varphi$  has  $\cos(\varphi) = -.2121$  and  $\sin(\varphi) < 0$ .
  - a) If  $\varphi$  is in standard position, which Quadrant contains the terminal side of angle  $\varphi$ ?
  - b) Find all the possible values, in radians, for  $\varphi$ . (Hint: Your answer should include an integer k.)
- **9.** Algebraically verify the identity.

$$1 - \sin y = \frac{\cos^2 y}{1 + \sin y}$$

- **10.** Let  $y = A \sin(Bx + C)$ , where A, B, and C are positive. Given the following information, find A, B, and C, and write out the correct expression for y.
  - Amplitude: 3
  - Period:  $\frac{p}{2}$
  - Phase Shift: -1

## **Part III**

- 11. Suppose that  $\alpha$ ,  $\beta$  are acute angles with  $\cos(\mathbf{a}) = \frac{3}{4}$  and  $\sin(\mathbf{b}) = \frac{2}{5}$ . Determine the **exact** value for  $\cos(\mathbf{a} + \mathbf{b})$ .
- 12. a) Use the sum identities to algebraically verify this identity:

$$\sin 2x = 2\sin x \cos x$$

- b) Let  $\mathbf{q}$  be an acute angle and  $\tan \mathbf{q} = \frac{1}{4}$ . Find the exact value of  $\sin (2\mathbf{q})$ . Calculator answers receive no credit.
- **13.** Which angles  $\mathbf{q}$ , in the interval  $[0^{\circ}, 360^{\circ})$ , satisfy  $\cos(2\mathbf{q}) = \frac{1}{2}$ ? Find the **exact** values, in degrees, of all four answers.
- **14.** Given that x is in the interval  $[0, 2\mathbf{p})$ , use algebraic methods solve the trigonometric equation  $\tan x = 2 \sin x$ . **Exact** answers are required.
- **15.** a) Convert the polar coordinates point  $\left(7, \frac{2\mathbf{p}}{3}\right)$  to rectangular coordinates. (Answer should be accurate to 2 decimal places.)
  - b) Change the polar equation  $r = -2 \sin q$  to rectangular form.
  - c) Change  $y = x^2$  to polar form.