MATH 150

AUTUMN 2001

Section 6.5, page 494

23 Find exact solution for θ in degrees:

$$2\sin^2(\theta) + \sin(2\theta) = 0, \text{ all } \theta$$

Proof. Let me begin with a comment: I found later today an easier proof - still, it's quite un-intuitive, and I maintain my claim that the chances of having this particular problem in the midterm is very low. OK, here's the proof now.

Express $\sin(2\theta)$ using the double-angle formula in terms of sin and cos of just θ:

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

We get, hence:

$$2\sin^2(\theta) + 2\sin(\theta)\cos(\theta) = 0$$

$$2\sin(\theta)(\sin(\theta) + \cos(\theta)) = 0$$

A product equal to zero means at least one of the factors is zero. We have the following two cases:

case 1: $2\sin(\theta) = 0$; we know that $\sin^{-1}(0) = 0^\circ$, so we can write

$$\theta_1 = 0^\circ$$

The second reference angle is the supplement of this angle:

$$\theta_2 = 180^\circ - 0^\circ = 180^\circ$$

Since the problem asks for "all θ" we finish this case by saying that the answer is $0^\circ + 360^\circ k$ and $180^\circ + 360^\circ k$ (the above two results plus a multiple of 360°).

case 2: $\sin(\theta) + \cos(\theta) = 0 \Rightarrow \cos(\theta) = -\sin(\theta)$. Here we are tempted to divide by $\sin(\theta)$and get tan ... but a bit of discussion is required: we cannot divide by zero (in general) so we MUST make sure that $\sin(\theta)$ is not zero for our problem. Let's see - if $\sin(\theta) = 0$ it means that $\cos(\theta) = \pm 1$, so the equality above cannot be satisfied; hence the sin is not zero.

Now divide by $\sin(\theta)$ and get

Date: 11/13/2001.

$$\tan(\theta) = -1$$

$$\tan^{-1}(-1) = -45^{\circ}$$

(use the calculator, if you cannot remember this). As said in class, for the tan case things are simple: you found your reference angle by using $\tan^{-1}$, and now just add a multiple of 180° to it (tan has a period of 180°, as you know, as opposed to sin and cos, which both have period of 360°). Since the problem (again) stated "all θ" just say $-45^{\circ} + 360^{\circ}k$ or even in words: "the above angle plus a multiple of 180°".

Done!

□