SOLUTIONS SAMPLE MIDTERM 2 AUTUMN 2000 AND SPRING 2001

MATH 150 AU01

Comment: no number computation is presented, but rather the method used ...

AU 2000

1.

(1) false: $\log(4x^3) = \log(4) + \log(x^3) = \log(4) + 3\log(x)$

(2) false: $2^x = e^{\ln(2)x}$

(3) false: $\log(x + y)$ cannot be changed

(4) true: positive cos axis and negative sin axis means 4th quadrant

(5) true: draw the reference triangle, which has opposite side's length of x and adjacent side's length of 1; hypothenusis' length is by Pythagora's Theorem equal to $\sqrt{1 + x^2}$ and hence $\sin(arctan(x)) = \frac{opp}{hyp} = \frac{x}{\sqrt{x^2+1}}$

(6) false: that's just ONE of the possible θs; in fact θ can equal $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$ as well, or a multiple of 2π added to those two (mind the question: the way it's put it implies that $\theta = \frac{\pi}{6}$ is the ONLY solution; that's what makes the answer false, even though that particular choice of θ works)

(7) false: $\sin^{-1}(x)$ is the INVERSE function, not the reciprocal of $\sin(x)$, which would be written then like this: $(\sin(x))^{-1}$

2.

if A is the initial amount of money we have to solve the equation: $(1 + 0.03)^t A = 2A \Rightarrow 1.03^t = 2 \Rightarrow t = \frac{\ln(2)}{\ln(1.03)}$

3.

$$1 - \log(x + 3) = \log(x)$$

$$1 = \log(x) + \log(x + 3) = \log(x(x + 3))$$

$$10^1 = 10^{\log(x(x+3))}$$

$$10 = x(x + 3) = x^2 + 3x$$

$$x^2 + 3x - 10 = 0$$

$$(x - 2)(x + 5) = 0 \Rightarrow x = 2, x = -5$$

Date: 11/14/2000.

Check answers, by plugging those two numbers back in the original equation: 2 works fine, -5 doesn't: $\log(-5+3) = \log(-2)$ doesn't exist, same $\log(-5)$. So the only answer is $x = 2$.

4.

(a) sin and cos negatives $\Rightarrow$ 3rd quadrant

(b) use calculator and get $\sin^{-1}(-.3598)$ which will be a negative number. If you try to compute cos of that angle you get a positive number, which is wrong. So, use the second choice, namely $180-$ that value, which now will work, and in fact will also be between 0 and 360 (the previous negative answer was not between those two).

5.

break the long side in two where the height meets it, and call the left piece a and the right piece b. then we have

$$\frac{a}{h} = \cot(\alpha)$$

$$\frac{b}{h} = \cot(\beta)$$

add those two together

$$\frac{a+b}{d} = \cot(\alpha) + \cot(\beta)$$

but $a+b$ make the whole side back together, hence $a+b=d$

6.

take the triangle which has as vertices: the observation point in the field, the point on the ground directly under Donald, Donald's foot. in this triangle we know the angle of 49 and the adjacent side of 74. the opposite side can be computed using tan:

$$\tan(49) = \frac{opp}{adj} \Rightarrow opp = \tan(49) \cdot 74$$

where in fact opp stands for the height of the building.

take a second triangle: observation point, point on the ground directly under Donald, Donald's tip of hand. again we know the angle of 59 and the adj=74. find opp

$$\tan(59) = \frac{opp}{74} \Rightarrow opp = \tan(59) \cdot 74$$

where now opp stands for height of building PLUS height of Donald. Substract the height of building found previously, and you're done.

7.

the left hand side is as simplified as possible, so let's concentrate on the right hand side

$\frac{\tan(t)+1}{\sec(t)} = \frac{\frac{\sin(t)}{\cos(t)}+1}{\frac{1}{\cos(t)}} = \frac{\sin(t)+\cos(t)}{\cos(t)} \cdot \frac{\cos(t)}{1} = \sin(t) + \cos(t)$

done

8.

(a) the y coordinate of the highest point gives the maximum of $\frac{1}{2}$ for this function, hence $A = \frac{1}{2}$

the period is twice $\frac{\pi}{2}$ since the function needs half the period to reach max from bottom and then another half to reach bottom again, hence period is π, and so $B = \frac{2\pi}{\pi} = 2$

for the shift, be warned that the function used is COS, not SIN ... cos starts in the max, which max is attained at $\frac{\pi}{2}$, hence shifted RIGHT, so $shift = -\frac{\pi}{2}$. *phase* $shift$, on the other hand, is negative the shift, so *phase* $shift = \frac{\pi}{2}$

(b) the formula is, hence: $\frac{1}{2}\cos(2(x - \frac{\pi}{2})) = \frac{1}{2}\cos(2x - \pi))$ ... everything is positive, so we're happy (always go to the right is the formula says $-C$ in there and try to go to the left when it's $+C$ whenever they ask you for POSITIVE values ... A and B will always be positive, C sometimes is negative ... eh, don't worry to much about it)

SP 2001

1.

(1) true: exponentials are always positive

(2) true: $x = e^{\ln(x)}$ whenever $\ln(x)$ exists (positive x), so squaring both sides we get the stated identity

(3) true: for every angle, the supplement ($180 - angle$ os $\pi - angle$ has the same sin value)

(4) false: you need ln to cancel the exponential with base e ... not log in base 10

(5) true: 90 degrees gives cos equal to 0, but then if you go around the unit circle a 180 degrees arc you get again 0 for cos, and another 180 brings you back, and so on ... same if you go backwards

(6) false: it's 4th quadrant

(7) $sin^{-1}(x)$ stands for INVERSE function, not reciprocal: $\csc(x) = (\sin(x))^{-1} = \frac{1}{\sin(x)}$

2.

solve the equation: $(1 + \frac{0.06}{4})^t \cdot 10000 = 20000 \iff 1.015^t = 2 \Rightarrow t = \frac{\ln(2)}{\ln(1.015)}$

3.

$$\log(x-7) + \log(x-4) = 1$$

$$\log((x-7)(x-4)) = 1$$

$$10^{\log(x^2-11x+28)} = 10^1$$

$$x^2 - 11x + 28 = 10$$

$$x^2 - 11x + 18 = 0 \iff (x-2)(x-9) = 0 \Rightarrow x = 2, x = 9$$

check the answers: 2 doesn't work, since plugging 2 back in the original equation yields $\log(2-7) = \log(-5)$ and $\log(2-4) = \log(-2)$ which do not exist. 9 on the other hand works perfectly. that's the only answer.

4.

(a) both cos and sin are negative $\Rightarrow$ 3rd quadrant

(b) find using calculator $\sin^{-1}(-0.12345)$ which yields a negative number. plugging this number in cos will yield a positive number though, not good (we expected negative cos) so let's use the other angle, the supplement of it ... this one will work; moreover, it's positive and less than 360, so we're done

5.

left hand side: $\sin(2x) = 2\sin(x)\cos(x)$

right hand side: $\tan(x)(1 + \cos(2x)) = \frac{\sin(x)}{\cos(x)}(1 + \cos^2(x) - \sin^2(x))$

we want to get rid of the denominator caused by the tan, so we would like to only have cos in the paranthesis, hence transform $\sin^2(x)$ into $1-\cos^2(x)$ and get

$\frac{\sin(x)}{\cos(x)}(1+\cos^2(x)-(1-\cos^2(x))) = \frac{\sin(x)}{\cos(x)}(1+\cos^2(x)-1+\cos^2(x)) = \frac{\sin(x)}{\cos(x)}\cdot 2\cos^2(x) = 2\sin(x)\cos(x)$

done

6.

draw the reference triangle with opposite side equal to x and hypothenusis equal to 1; adjacent side will be $\sqrt{1-x^2}$ and so

$$\tan(\sin^{-1}(x)) = \frac{opp}{adj} = \frac{x}{\sqrt{1-x^2}}$$

7.

$$f(2) = Ca^2 = 22.5$$

$$f(8) = Ca^8 = 16402.5$$

divide the second to the first and get

$$a^6 = \frac{16402.5}{22.5} = 729 \Rightarrow a = \sqrt[6]{729} = 3$$

find C:

$$C \cdot 3^2 = 22.5 \Rightarrow 9C = 22.5 \Rightarrow C = 2.5$$

8.

(a) amplitude is 3

period is π (interval from bottom to max is $\frac{\pi}{2}$ and we need one more $\frac{\pi}{2}$ to get back to the bottom.

since we're dealing with a COS, and cos starts at the max, we notice it's shifted $\frac{\pi}{2}$ to the right, hence a $shift = -\frac{\pi}{2}$, so $phase\ shift = \frac{\pi}{2}$.

(b) since the thing has -5 for A we must be careful: draw the graph first, and notice that it starts going DOWN, not up, as it is supposed to do; this will influence the $shift$ and $phase\ shift$

amplitude is still 5

period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$ (where 2 is the coefficient of x)

rewrite the function now: $-5\sin(2(x+1.5))$ ... one candidate for the shift would be 1.5 (a left shift for sin, and so phase shift would be -1.5 ... this function's starting point is at -1.5 hence); but, as said, the -5 in front changes everything: look at the graph, since the graph starts going up not at -1.5, but before (or after) HALF A PERIOD, so the actual $shift = 1.5+\frac{\pi}{2}$ and so $phase\ shift = -1.5-\frac{\pi}{2}$

done