

May 1, 2000

Instructions: The examination is closed book, other than one page of formulae. Please print your name on the exam. YOU MUST SHOW ALL YOUR WORK TO GET FULL CREDIT.

1. (20 Points) T or F (If true just write true; if false give counterexample or correct calculation.)

(a) If sequences $\{a_n\}$ and $\{b_n\}$ are divergent, then sequence $\{a_n + b_n\}$ is divergent.

(b) If $\sum a_n$ is divergent, then $\sum |a_n|$ is divergent.

(c) If $\sum_{n=0}^{n=+\infty} C_n(-2)^n$ diverges, then $\sum_{n=0}^{n=+\infty} C_n 3^n$ diverges.

(d) If sequence $\{a_n\}$ converges, then $\sum a_n$ converges.

2. (15 Points) Calculate the limits.

(a) $\lim_{n \rightarrow \infty} \frac{n^2 - \sqrt{n}}{4 - n^2}$

(b) $\lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\ln n}$

(c) $\lim_{n \rightarrow \infty} \frac{n!}{n^n}$

3. (15 Points) Find the Taylor series of the function about $a = 2$.

$$f(x) = \ln x$$

4. (15 Points) Find the first four non-zero terms of the Maclaurin series for $f(x) = x \sin(x^2)$. Use this to find the eleventh derivative of $f(x)$ at $x = 0$.

5. (15 Points) Given

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots + x^n + \cdots$$

where $|x| < 1$. Use it to calculate the sum of the series

$$\frac{1}{3} + \frac{2}{3^2} + \frac{3}{3^3} + \cdots + \frac{n}{3^n} + \cdots$$

6. (20 Points) Find the radius of convergence and interval of convergence for the power series

$$20 \quad \sum_{n=0}^{n=+\infty} \frac{x^n}{n \ln n}$$