MIDTERMS' ANSWERS

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Midterm 2

1. (a) the equation giving the r is:

$$r^2 - 4r + 4 = 0$$

Double solution - $r = 2$.

$$y = C_1 \cdot e^{2t} + C_2 \cdot te^{2t}$$

(b) the equation is now:

$$r^2 - 4r + 13 = 0$$

Complex roots: $r = 2 \pm 3i$

$$y = C_1 \cdot e^{2t} \cos(3t) + C_2 \cdot e^{2t} \sin(3t)$$

(c) the equation is now:

$$2r^2 + 8r + 6 = 0$$

which has as roots: $r = -3$, $r = -1$.

$$y = C_1 \cdot e^{-3t} + C_2 \cdot e^{-t}$$

2. Solve the homogeneous part first - it is almost the problem in part 1(a)

$$y_H = C_1 \cdot e^{-2t} + C_2 \cdot te^{-2t}$$

For the non-homogeneous part let's take the constant first.
Guess: $y = A$.

$$A'' + 4A' + 4A = 5 \Rightarrow 4A = 5 \Rightarrow A = \frac{5}{4}$$

Take now the exponential.
Guess: $y = Be^{-3t}$

$$9Be^{-3t} + 4 \cdot (-3)Be^{-3t} + 4Be^{-3t} = 3e^{-3t} \Rightarrow B \cdot e^{-3t} = 3e^{-3t} \Rightarrow B = 3$$

Complete solution:

$$y = C_1 \cdot e^{-2t} + C_2 \cdot te^{-2t} + \frac{5}{4} + 3e^{-3t}$$

3. Solve the homogeneous part first; complex roots: $R^2+1=0 \Rightarrow r=\pm i$. Hence

$$y_H = A\cdot\sin(t) + B\cdot\cos(t)$$

Guess for the non-homogeneous part: $y = C\cdot\cos(t) + D\cdot\sin(t)$. Checking this will give us 0 in the left hand side though!!

New guess: $y = (C\cdot\cos(t) + D\cdot\sin(t))\cdot t$

$$y' = (-C\cdot\sin(t) + D\cdot\cos(t))\cdot t + (C\cdot\cos(t) + D\cdot\sin(t))$$

$$y'' = (-C\cdot\cos(t) - D\cdot\sin(t))\cdot t + 2(-C\cdot\sin(t) + D\cdot\cos(t))\cdot 1$$

$$y'' + y =$$
$$= (-C\cdot\cos(t) - D\cdot\sin(t))\cdot t + 2(-C\cdot\sin(t) + D\cdot\cos(t))\cdot 1 + (C\cdot\cos(t) + D\cdot\sin(t))\cdot t =$$
$$= 2(-C\cdot\sin(t) + D\cdot\cos(t))$$

and this must equal $3\sin(t)$, hence the coefficient for cos is 0; $-2C\cdot\sin(t) = 3\sin(t) \Rightarrow C = -\frac{3}{2}$.

Complete general solution:

$$y = A\cdot\sin(t) + B\cdot\cos(t) - \frac{3}{2}\cos(t)\cdot t$$

Plug-in initial conditions: $y(0) = B = 1$; $y'(0) = -A - \frac{3}{2} = 0 \Rightarrow A = -\frac{3}{2}$.

4. Homogeneous, non-constant coefficients, hence reduction of order method.

$$y_2 = v \cdot y_1 = v \cdot t$$

$$t^2(v'' \cdot t + 2v' \cdot 1) - 2t(v' \cdot t + v \cdot 1) + 2vt = 0$$
$$t^3 v'' = 0$$

Divide by t^3:

$$v'' = 0$$
$$v' = A$$
$$v = At + B$$

$$y_2 = v \cdot t = At^2 + Bt$$

A second, linearly independent solution would, then, t^2.

5. Write in standard form:

$$y'' - \frac{2}{t^2}y = 3 - \frac{1}{t^2}$$

Nonhomogeneous, non-constant coefficients, hence variation of parameters.
Find Wronskian for the two particular solutions of the homogeneous equation:

$$\begin{vmatrix} t^2 & 2t \\ t^{-1} & -t^{-2} \end{vmatrix} = -1 - 2 = -3$$

Write the formula for the non-homogeneous solution:

$$y = -t^2 \int \frac{t^{-1} \cdot (3 - \frac{1}{t^2})}{-3} + t^{-1} \int \frac{t^2 \cdot (3 - \frac{1}{t^2})}{-3} =$$
$$= \frac{1}{3}t^2(3\ln(t) - \frac{1}{2}t^{-2}) - \frac{1}{3}t^{-1}(t^3 - t)$$

General solution:

$$y = c_1t^2 + c_2t^{-1} + \frac{1}{3}t^2(3\ln(t) - \frac{1}{2}t^{-2}) - \frac{1}{3}t^{-1}(t^3 - t) =$$
$$= c_1t^2 + c_2t^{-1} + t^2\ln(t) - \frac{1}{6} - \frac{1}{3}t^2 + \frac{1}{3}$$