SOLUTIONS CHAPTER 5.2

MATH 548 - SP00

$\boxed{2}$

Proof. We know that x is continuous on $\mathbf{R}$, hence $\underbrace{x * x * ... * x}_{n\ times}$ is also continuous (5.2.1) on $\mathbf{R}$. f is continuous on A, so if we take the composition of these two functions, which results into f^n, we get a continuous on A function.

□

$\boxed{4}$

Proof. We know that $[x]$ is continuous everywhere, except integer points (lateral limits don't coincide ... check it!); x is continuous on the whole $\mathbf{R}$, hence we know for sure that $f = x - [x]$ is continuous everywhere, except integer points. Now, if f were continuous on at least one integer point we'll get that $[x] = x - f(x)$ is continuous there (since both terms are continuous), but this is absurd. So no integer point will do.

□

$\boxed{6}$

Proof. Let's take the sequential approach: take a sequence $(x_n)_{n\in\mathbf{N}}$ which converges to c $\Rightarrow f(x_n) \to b$ (hypothsesis); g is continuous at b, hence any sequence that converges to b, through g, will become a sequence converging to $g(b)$ (continuity). Hence we have, for that (arbitrary chosen) $(x_n)_n$ that $\lim g(f(x_n)) = g(b)$, hence

$$\lim_{x\to c} g \circ f = g(b)$$

□

$\boxed{7}$

Proof. Define f as follows: $f(x) = 1$ if $x \in \mathbf{Q}$; $f(x) = -1$ if $x \in \mathbf{R}\backslash\mathbf{Q}$. As we did in a previous homework, we can prove this f is NOT continuous anywhere. But if one takes $|f| = 1$ this is continuous.

□

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$\boxed{9}$

Proof. The trick would be to find for any c, a sequence of the form $\frac{m}{2^n}$ that will converge to c. Idea is, we can write any number (even with decimals) as a sum of powers of 2 (or, as it's called, write the number is base 2). Take an approximation (or a partial sum of that sum of powers of 2) ... and if one adds them, with common denominator being obviously a power of 2, we get a number of the above form. Now, take all approximations, and they form a sequence, and it converges to our c, and through f they give us 0, hence f(c)=0 - because of continuity. □

$\boxed{11}$

Proof. Take $h = f - g$ and compare with problem 8, chapter 5.1 ($x \in S \Rightarrow h(x) = f(x) - g(x) \geq 0$ etc). □

$\boxed{12}$

Proof. Using the sequential approach, we have that, for any c and $y_n \to c$, $y_n - c + x_0 \to c - c +_0 = x_0$ hence $f(y_n - c + x_0) \to f(x_0) \Rightarrow f(y_n) = f(y_n - c + x_0) + f(c - x_0) \to f(x_0) + f(c - x_0) = f(c)$ hence f is continuous at c. □

$\boxed{15}$

Proof. For any x, sup(x)=$f(x)$ if $f(x) \geq g(x)$ and $g(x)$ if $f(x) < g(x)$. Let's see what happens to the formula in both these cases: for the first one we get

$$\frac{1}{2}(f(x) + g(x)) + \frac{1}{2}(f(x) - g(x)) = f(x)$$

and for the second we get

$$\frac{1}{2}(f(x) + g(x)) + \frac{1}{2}(g(x) - f(x)) = g(x)$$

Exactly what we needed. Now, adding, dividing by 2 and composing continuous in c functions with the absolute value function (which is continuous in its own right everywhere) give us continuous in c functions. Hence sup is continuous in c. □