

TorusTrek

In some old computer games¹ there are a few space ships moving on the screen, along with various other objects. When an object (ship, bullet, asteroid, etc.) leaves the screen to the right it reappears on the left, as if the right and left edges of the video screen were connected together. In Figure 1, our ship is shooting out a stream of bullets, eventually hitting the asteroid.

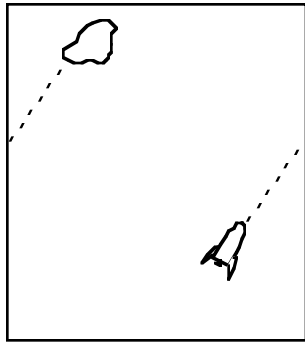


Fig. 1

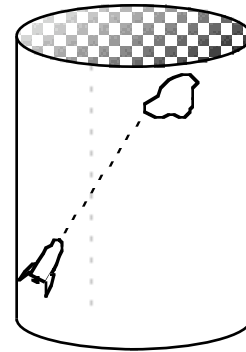


Fig. 2

Imagine a paper copy of the screen, and glue its right and left edges together to form a cylinder. This cylindrical version of the screen in Figure 2 explains the “wrap-around” effect for the left and right edges. On the cylinder there is no sudden jump of bullets from the right edge to the left: the bullets move naturally. This phenomenon reminds me of world maps hanging on schoolroom walls. The western tip of Alaska on the far left of the map seems distant from the eastern tip of Russia on the far right, but looking at a globe we see that those spots are actually very close to each other.

Another way to understand the geometry of this computer screen is to “unroll” that cylinder over and over, making a long strip containing many identical copies of the original screen. When the ship shoots a bullet, each of the identical ships on the strip simultaneously shoots a bullet, and those bullets all travel smoothly without any unnatural jumps. The boundaries of the original screen appear as evenly spaced vertical separators on this strip: they are an artificial choice, not part of the natural geometry. We could cut the cylinder along a different vertical line to get a somewhat different screen representing the same picture. Of course there is really only one ship there, represented as a long line of identical images.

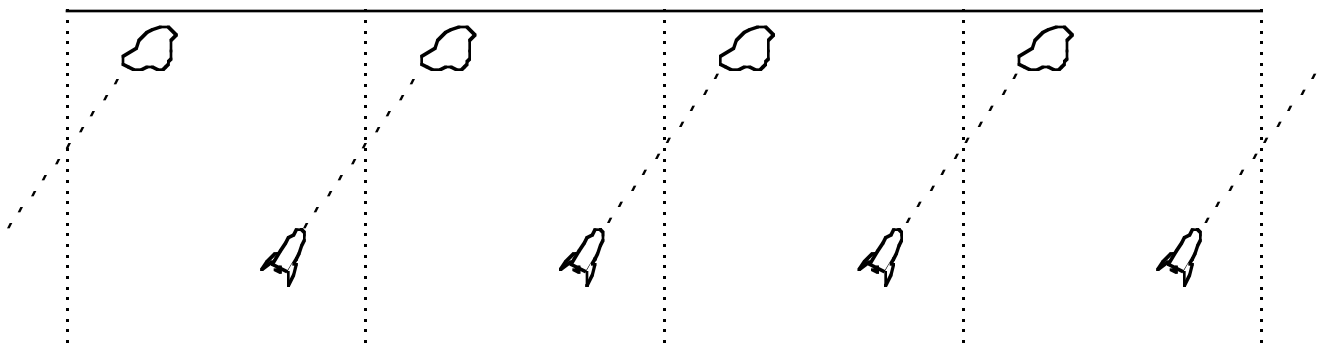


Fig. 3

¹If you haven't seen these ancient games yourself, imagine your ancestors playing them.

Back in the original game, when a bullet leaves at the top of the video screen it reappears at the bottom, as if the top and bottom edges of the screen were connected together. In Figure 4 the stream of bullets has split the asteroid in two and continues on. (Luckily, those bullets missed the ship, but they came so close that the gunnery sergeant was reprimanded.)

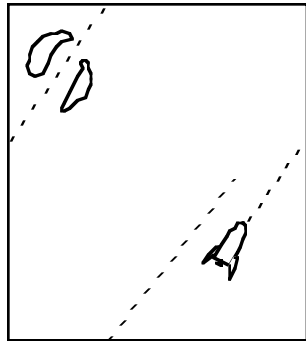


Fig. 4

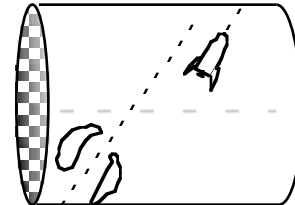


Fig. 5

As before let's consider a paper copy of the video screen, this time gluing the top and bottom edges together. This cylindrical version of the screen in Figure 5 shows the "wrap-around" effect for the top and bottom edges. Now there is no sudden jump of bullets from the top to the bottom (although there is still a jump from left to right in the cylinder picture). This cylinder could also be unrolled into a long vertical strip of identical copies of the screen.

We glued vertical edges one time and we glued horizontal edges the other time. Now we want to glue both pairs of edges at the same time. Some stretching of the surface seems required to actually do this. Let's start with the cylinder in Figure 5, which has two circular edges, one on the left and one on the right. Stretch that cylinder and smoothly bend it around so that those two circular edges are facing each other, as in Figure 6.

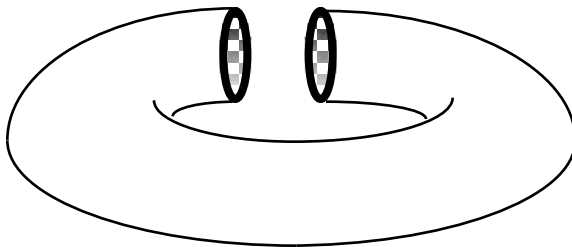


Fig. 6

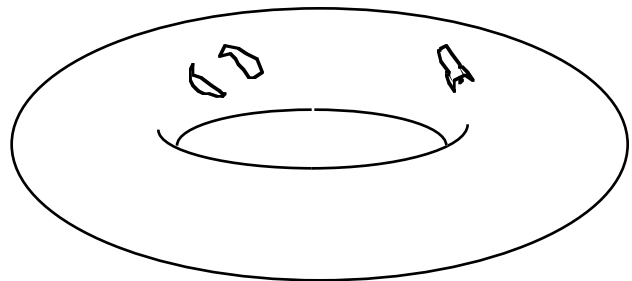


Fig. 7

Move those circular edges together and attach them to form a closed donut-shaped object as in Figure 7, with our space ship and asteroid picture drawn on its surface. That surface is called a torus.² The bullets don't make unnatural jumps across any edges: the torus has no edges at all! Cutting it along a vertical circle leads to Figure 6 and the horizontal cylinder of Figure 5. We could instead cut a horizontal circle around the hole of the torus, and then straighten the torus out vertically to obtain the cylinder in Figure 2.

Another way to understand the geometry of the torus is to "unroll" it over and over in both directions, making a plane covered by many identical copies of the flat screen in Figure 4. Every point of this screen is represented in Figure 8 by a whole two dimensional array of points equally spaced across the plane. (The six copies of the picture here are supposed to be repeated

²The plural form of torus is "tori", like cactus & cacti and octopus & octopi.

indefinitely in all directions.) There is “really” only one ship and one (split) asteroid and one bullet stream in this unrolled view.

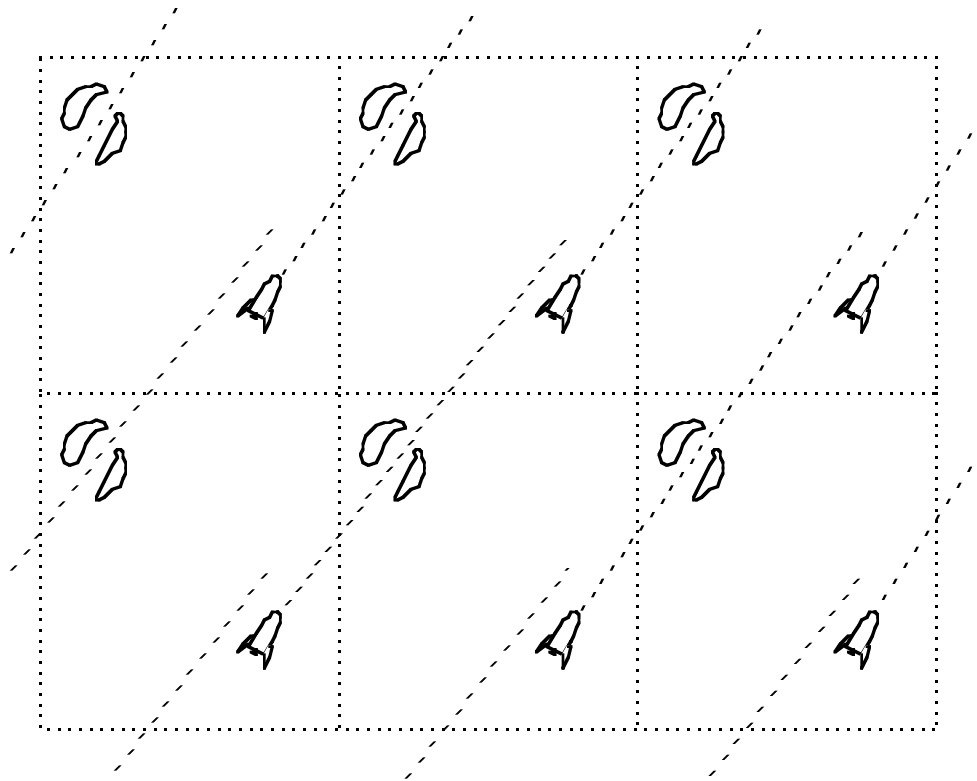


Fig. 8

We “unrolled” that screen into those multiple copies in order to demonstrate that each bullet travels in a straight line without any jumps. Those jumps in the original game from right to left and from up to down are not really jumps at all. They result from the artificial way the torus was represented as a flat video screen.

This article explains that the original computer game is actually played on a torus, not on a plane. Now whenever you see a rectangular wallpaper pattern repeated over and over you can say to yourself: “That’s really an unrolled version of a torus.”

Food for thought:

- (1) Suppose a stationary ship shoots out a bullet in this torus universe. Can that unfortunate gunnery sergeant shoot out a bullet which will never return to hit the ship? (Assume that there are no objects that can stop or deflect the bullet.)
- (2) A 3-dimensional torus is harder to visualize. Imagine a solid box where the opposite walls are glued together. Shoot through the left wall and your bullet enters the same room from the right. Jump through the front wall and you re-enter into the room from the back. Pound a nail into the floor and the tip of the nail comes in through the ceiling. This room can be “unrolled” into a 3-dimensional array of many identical copies of this room, and the path of each bullet becomes a straight line without jumps. However it requires four spatial dimensions to really see this 3D torus as a single object with all of its faces glued together correctly. Most people can’t visualize it directly, but can work with it by thinking of solid boxes.