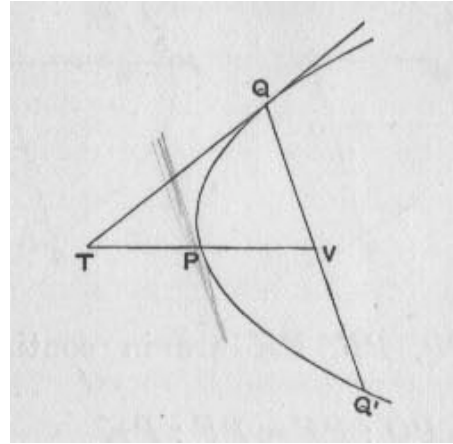
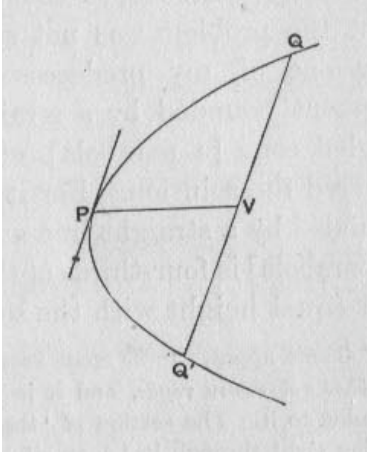


Proposition 1.

If from a point on a parabola a straight line be drawn which is either itself the axis or parallel to the axis as PV, and if QQ' be a chord parallel to the tangent to the parabola at P and meeting PV in V, then $QV = VQ'$. Conversely if $QV = VQ'$ the chord QQ' will be parallel to the tangent at P.

**Proposition 2**

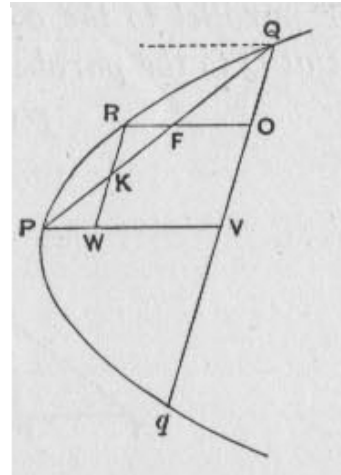
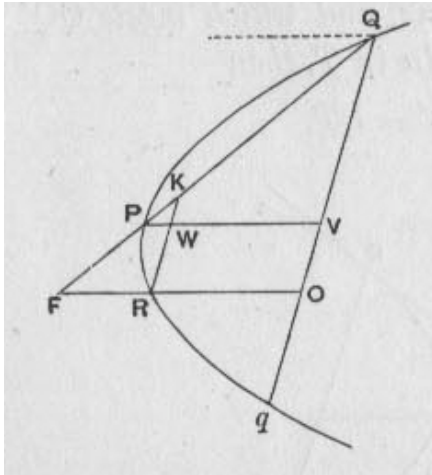
If in a parabola QQ' be a chord parallel to the tangent at P and if a straight line be drawn through P which is either itself the axis or parallel to the axis, and which meets QQ' in V and the tangent at Q to the parabola in T then, $PV = PT$.

Proposition 3

If from a point on a parabola a straight line be drawn which is either itself the axis or parallel to the axis, as PV and if from two other points Q, Q' on the parabola straight lines be drawn parallel to the tangent at P and meeting PV in V, V' respectively, then $PV : PV' = QV^2 : Q'V'^2$

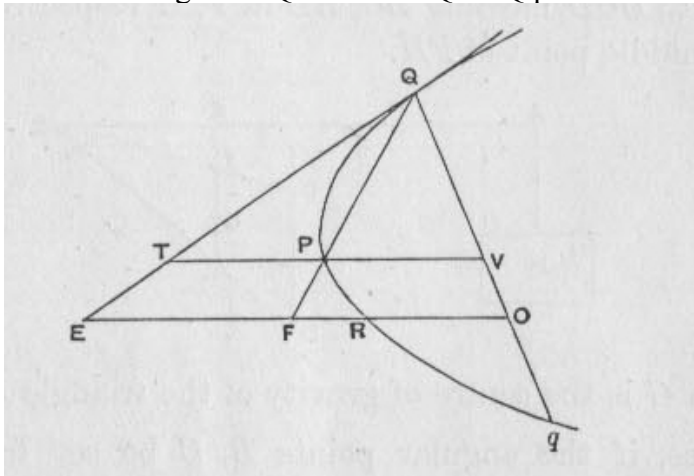
Proposition 4

If Qq be the base of any segment of a parabola, and P the vertex of the segment and if the diameter through any other point R meet Qq in O and QP (produced if necessary) in F, then $QV : VO = OF : FR$



Proposition 5.

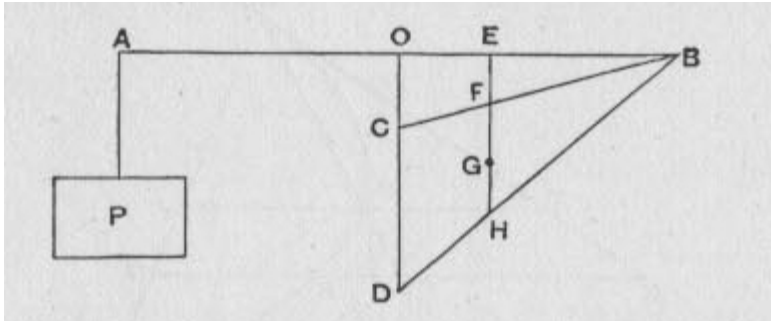
If Qq is the base of any segment of a parabola, P the vertex of the segment, and PV its diameter and if the diameter of the parabola through any other point R meets Qq in O and the tangent at Q at E then $QO : Qq = ER : RO$



Propositions 6, 7

NOTE: proposition 6 is for a right angle and 7 for an obtuse angle, but the same proof suffices for both.

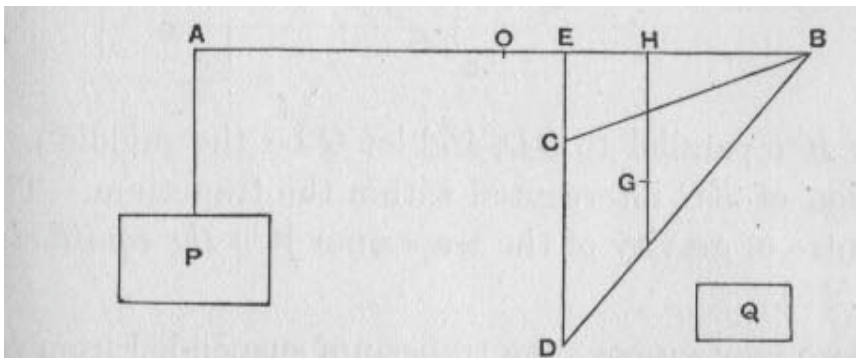
Suppose a lever AOB placed horizontally and supported at its middle point. Let a triangle BCD in which the angle C is right or obtuse be suspended from B and O so that C is attached to O and CD is in the same vertical line with O . Then if P be such an area as when suspended from A , will keep the system in equilibrium P is equal to one third the area of triangle BCD .



Propositions 9, 10

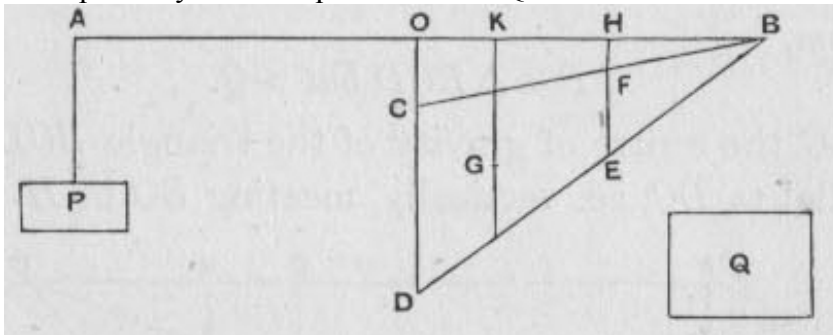
Suppose a lever AOB be placed horizontally and supported at its middle point O. Let a triangle BCD, right angled or obtuse-angled at C be suspended from the points B, E on OB the angular point C being so attached to E that the side CD is in the same vertical line with E. Let Q be an area such that $AO : OE = \square BCD : Q$

Then if an area P suspended from A keeps the system in equilibrium $Q < P < \square BCD$



Propositions 10, 11

Suppose a lever AOB be placed horizontally at O, its middle point. Let CDEF be a trapezium which can be so placed that its parallel sides CD, FE are vertical, while C is vertically below O, and the other sides CF, DE meet in B. Let EF meet BO in H, and let the trapezium be suspended by attaching F to H and C to O. Furthermore let Q be an area such that $AO : OH = \text{area}(CDEF) : Q$. Then if P be the area which when suspended from A keeps the system in equilibrium $P < Q$



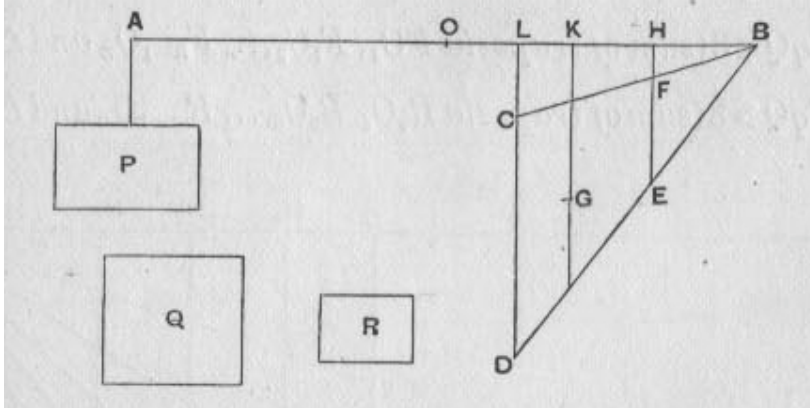
Propositions 12, 13

If the trapezium CDEF be placed as in the last propositions, except that CD is vertically below a point L on OB instead of being below O, and the trapezium is suspended from L, H and suppose that Q, R are areas such that

$$AO : OH = \text{area}(CDEF) : Q$$

$$AO : OL = \text{area}(\text{CDEF}) : R$$

Then if P is an area suspended from A that keeps the system in equilibrium. $Q>P>R$



Propositions 14, 15

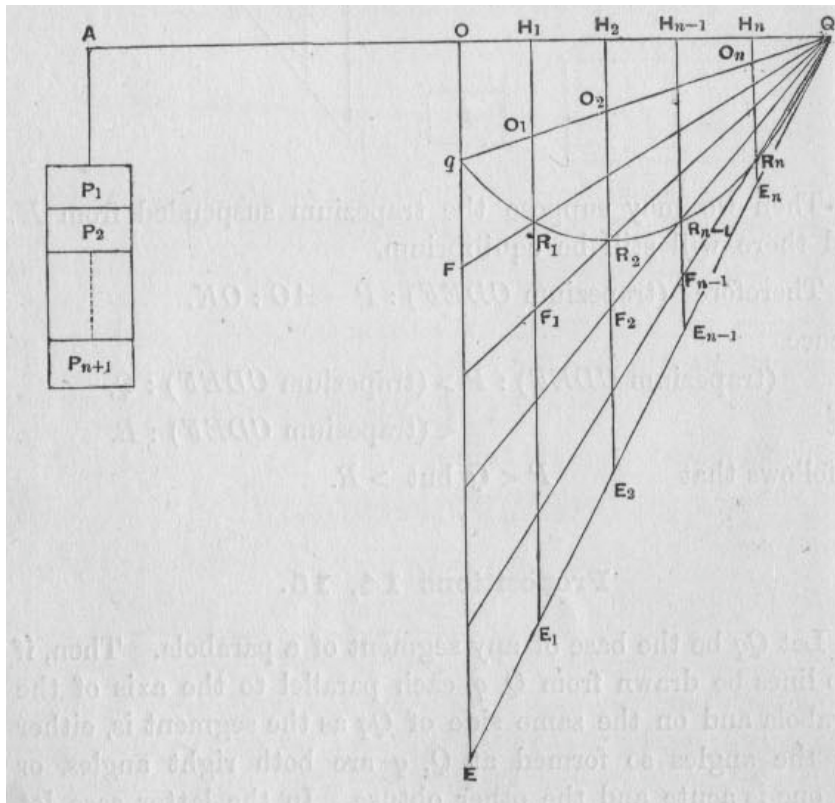
Let Qq be the base of any segment of a parabola. Then if two lines be drawn from Q, q each parallel to the axis of the parabola and on the same side of Qq as the segment is, the angles at Q and q are both right or the angle at q is obtuse and the one at Q acute.

Divide Qq into any number of equal parts at the points O.... connect these points to the tangent at Q in E.... and the parabola itself at q R....

It is required to prove that:

$$\|EqQ\| < 3[(\|F_{n+1}O_n\|) + \|E_nO_nQ\|]$$

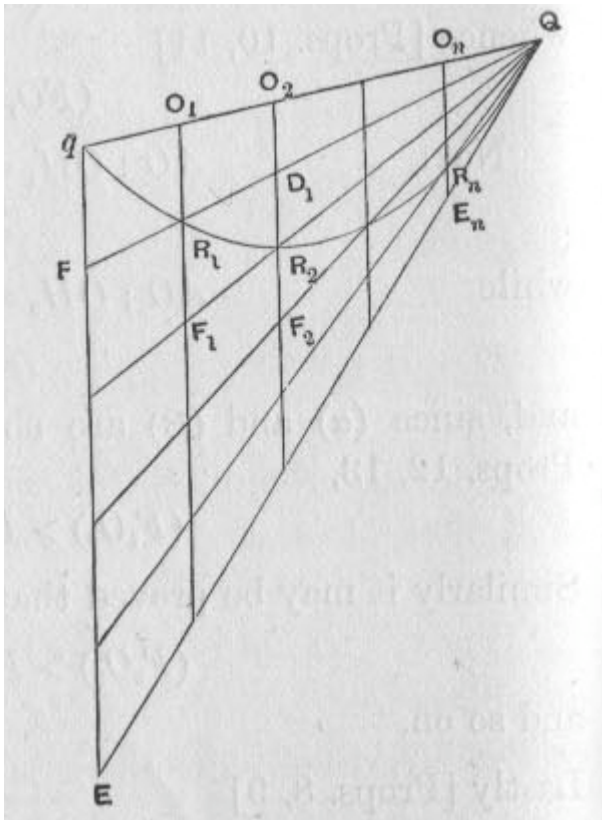
$$EqQ > 3[(R_{n+1}O_n) + R_nO_nQ]$$



Proposition 16.

Suppose Qq to be the base of a parabolic segment, q being not more distant than Q from the vertex of the parabola. Draw through q the straight line qE parallel to the axis of the parabola to the tangent at Q in E . It is required to prove that

$$\text{area}(\text{segment}) = \frac{1}{3} \square EqQ$$



Proposition 17

It is now manifest that the area of any segment of a parabola is $\frac{4}{3}$ of the triangle which has the same base as the segments and equal height