Math 105

Midterm Exam Review

Autumn Quarter 2009

Overview

The exam will cover all that is done in class through the end of Chapter 4 (through Monday, October 26).

FIRST, you should study the activities, homework problems, and quizzes.

- SECOND, you should study the text's practice problems (as well as problems and activities that have not been assigned) toward deepening your knowledge.
- THIRD, some additional "practice" problems are given below. Some of these are more challenging and are intended to stretch your understanding.

There will be 5 questions on the exam, each somewhat similar to our weekly quiz questions in form. Assume that you will need to explain your solutions unless otherwise stated on the exam. Each problem will be graded based on the same 10-point rubric used to grade the homework and quizzes.

CALCULATORS WILL NOT BE ALLOWED ON THE EXAM, but you may use a calculator on any of the "additional" problems below.

Here are some guidelines from chapter to chapter. Note that this is not an exhaustive list of topics we've experienced.

Chapter 2

1. Know the characteristics of the place value system of numeration (e.g., finite set of symbols used, one symbol for each place, each place has a specific value for what you are counting, counting via "bundling rules", etc.).
2. Know how to physically represent quantities by our place value system (e.g., Base Ten blocks, toothpicks and rubber bands, number lines, etc.). Know to establish what each type of object is worth (it's "value" in the place value system).
3. Know how to compare quantities represented by our place value system. Be ready to employ techniques and defend those techniques using physical objects.
4. Know how to represent positive and negative numbers using a number line, chips, or "real life" contexts.

Chapter 3

1. Know the meaning and significance of the whole, denominator, and numerator of a fraction. Know how pictures should be labeled or described to avoid ambiguity.
2. Know how to argue that equivalent fractions represent the same quantity. Similarly know how to show what really happens when a fraction is "reduced to lowest terms".
3. Know how to justify techniques for comparing fractions.
4. Know how to represent and solve all kinds of "percent" problems.
5. Know how to use pictures to represent, solve, and explain all of the above.

Chapter 4

1. Understand the meanings and different interpretations of "addition" and "subtraction" (e.g., take-away, missing addend, comparison).
2. Know how to analyze and write valid story problems: paying attention to the set-up, question, and units.
3. Be ready to explain and defend the standard and alternative algorithms for addition and subtraction of quantities written in the place value system (including integers) and as fractions.
4. Be ready to defend or refute arguments made by students for setting up computations for story problems (involving material from chapters 2, 3, and/or 4) or for computational techniques.
5. Know pitfalls of adding ratio amounts (e.g., when presented as percents).
6. Know how to represent, solve, and explain "percent increase and decrease" problems using pictures.
7. Know how to explain reasons for the commutative and associative properties of addition. Know how each contributes to and justifies algorithmic and mental math calculations.

Extra Problems

1. Argue why we can "cancel the zeroes" in simplifying $\frac{78000}{31000}$, but we cannot "cancel the threes" in simplifying $\frac{78333}{31333}$.
2. Use a picture to argue that $\frac{5}{4} = \frac{15}{12}$.
3. At a small college of 1000 students, there are only two intramural sports: football and softball. 600 students play softball and 750 students play football. From this information,

a. What is the least number of students who are playing both sports?
b. What is the most number of students who are playing both sports?
c. What is the most number of students who are playing intramural sports?
d. What is the least number of students who are playing intramural sports?
e. If there are 400 students playing both sports, how many students play intramural sports?

4. If the average tuition at private universities is $\frac{13}{9}$ of the average tuition at public universities and the average tuition at public universities is $12600, use a picture to find the average tuition at private universities.
5. John has spent a fifth of his life as a boy growing up, another one-sixth of his life in college, one-half of his life as a bookie, and has spent the last six years in prison. How old is John now? Use only concepts of and reasoning about addition and fractions (i.e., no multiplication or division) to solve this problem.

6. Daryl is having trouble subtracting mixed numbers. What might be causing his difficulty? Here is a sample of his work:

$$\begin{array}{r} 3\frac{2}{5} = 2\frac{12}{5} \\ -\ \frac{3}{5} = \frac{3}{5} \\ \hline 2\frac{9}{5} = 3\frac{4}{5} \end{array}$$

7. A recipe for cookies will prepare enough for three-sevenths of Mrs. Jordan's class of 28 students. If she makes three batches of cookies, how many extra students can she feed?
8. Sally thinks .0037 < .00086743987 because the second numeral is larger. Help Sally without using "rules" of symbolic place value.
9. The weight, rounded to the nearest hundredth of a gram, of a particular sample of ice tea mix, was 28.67 grams. If the actual weight was a number of grams that had 4 decimal places after the decimal point, what was the minimum and maximum possible actual weights of the mix?
10. If $\frac{5}{7}$ of a cup of orange juice satisfies your daily requirement of Vitamin C, what fraction of the daily requirement of Vitamin C would be satisfied by 2 cups of orange juice? Use a picture to solve the problem.
11. If you have a line segment that is $\frac{2}{3}$ inches long, describe how to remove or add a portion to the initial line segment to create a line segment that is $\frac{6}{5}$ inches long.

12. Show how to calculate $3\frac{2}{5} - 1\frac{2}{3}$ in two different ways without a calculator. In each case, express your answer as a mixed number and explain why your method is legitimate.
13. John and Anne want to solve 4.23 – 1.97 by first solving 4.23 – 2.00 = 2.23. John says we now need to *add* .03 to 2.23 while Ann says we need to *subtract* .03 from 2.23. Who is right (if either) and why?
14. Using chips and/or "mailing" problems (as described in lecture), describe why it makes sense to define (5) – (-4) to be the same as 9. Use both the take-away and missing addend models of subtraction.
15. Use a number line to find two numbers between -1.02 and -.987. Be sure to label the larger tick marks and state the distance between the smaller tick marks. The distance between adjacent tick marks must be a power of ten (i.e., 100, 10, 1, $\frac{1}{10}, \frac{1}{100}$, etc.). Finally, be sure that -1.02 and -.987 both lie precisely on tick marks.
16. From the text:
    2.2 #5
    2.4: #7, 16
    2.5 #5
    3.1 #7, 21
    3.3: #18
    3.5 #20, 21ab
    4.2 #12, #15
    4.3 #17
    4.4 #3
    4.5 #11, 13
    4.6 #4
17. You are in a checkout line and, when you get to the front, you find that your cashier is a friendly man named Tarzan. He seems to be bent on being as helpful as can be, but talks as if he has been raised in the jungle. He has been taught to run the register, but appears to have the mathematical sophistication of a small child. Some of his statements crack you up because the math they lack is so ingrained in you. Try to figure out which mathematical property (s) of addition prevent "normal" cashiers from making the following statements:

    a. The box has nothing in it, but when I ring it up, your total may change.

    b. Would you like me to ring up your ice cream before or after your cake? Your decision might make a big difference in the total, you know.

    c. Do you want me to ring up subtotals so you can keep track of what you're spending? You know, the way I choose to group the items in subtotals will affect your total cost.

18. Justify each step in the following algorithm by either, place value concept, commutative property of addition, associative property of addition, or basic fact (i.e.,

look at what has changed from one step to the next and answer "what allows us to say they're the same quantity?"):

$24 + 38 = (20 + 4) + (30 + 8)$
$= 20 + [4 + (30 + 8)]$
$= 20 + [(30 + 8) + 4]$
$= 20 + [30 + (8 + 4)]$
$= (20 + 30) + (8 + 4)$
$= 50 + 12$
$= 50 + (10 + 2)$
$= (50 + 10) + 2$
$= 60 + 2$
$= 62$

19. A clothing store advertised a coat at a 15% discount. The original price was $115 and the stated sale price was $100. Was the price consistent with the ad? Explain.
20. Describe, in terms of the meanings of *whole, numerator*, and *denominator*, why we need a "common denominator" when writing the sum of two fractions as one fraction.
21. For each of the following, calculate (exactly) mentally without using standard algorithms. Describe your thoughts.
    a) 479 + 97
    b) 135% of 80.
22. For each of the following, illustrate using the given technique.
    a) (-13) – (-7) using chips.
    b) 253 – 6.5 using base ten blocks.
23. Make up your own "story problems" that are set up by the following:
    a) (-3) – (5)
    b) 34 – 19 (missing addend)
24. A store reduced the price of a computer by 20% and sold it for $1760. How much did the computer originally sell for?
25. During the Red Sox's last weekend series, 46% of those attending Saturday's game were women and 42% of those attending Sunday's game were women. Were there more women in the stadium on Saturday or Sunday?

Numerical Answers to "Extra" Problems

3. a. 350 students, b. 600 students, c. 1000 students, d. 750 students, e. 950 students

4. $18200

8. 45 years old.

7. 8 more students

9. Min = 28.6650 grams, max = 28.6749 grams

10. $\frac{14}{5} = 2\frac{4}{5}$ daily values

17. b. Additive Identity c. Commutative of addition d. Associative of addition

18. In order of lines, beginning with first equals sign: place value; associative of addition; commutative of addition; associative of addition; associative of addition; addition; place value; associative of addition; place value; addition.

19. No. Discount is 15% of the *original* price. Thus, discount = .15(115)=$17.25 and new price = $97.75.

22. a. *Take-Away*: We have 13 red (negative) chips. If we take away 7 of those chips, what are we left with? (6 red chips = -6).

Missing Addend: We have 7 red (negative) chips. Our goal is to have 13 red chips. What do we need to join with what we have to achieve our goal? (6 red chips = -6).

24. $2200

25. We don't have enough information. We would need the actual attendances (i.e., what the percentages are *of)* (Moral: Percentages are ratios and only give "relative sizes", not actual)