

Fractions Without Rules and Algebra Without Letters

In the problems below, your task is to find a way to reason toward the solution without using the rules and methods that you have practiced. In particular, you may not use letters, set up a proportion, or invert and multiply. You may not use the terms “common denominator” or “equivalent fraction.” Instead you should reason from the context and from the numbers given. Your solution should make sense to someone who does not know the standard procedures and terms.

If you find it difficult to see other possibilities, give yourself a chance: draw a picture, make a diagram, reason with easier numbers, or make a guess and check it. Try reasoning backwards. If you are still stuck, solve the problem using your well-rehearsed method and then ask whether the answer or your solution helps you think of alternative approaches.

1. There are 200 fish in a pond; 98% of them are guppies. How many guppies need to be added to the pond in order to raise the percentage of fish that are guppies to 99%?
2. The father of a family of 4 children baked a pan of brownies one evening and left them on the kitchen counter to cool over night. The following morning, Billy came downstairs first and ate some of the brownies. Hannah followed Billy and ate $\frac{1}{3}$ of what was left. The twins, Rachel and Lindsey, were last to come down and they took half of what was left in the pan and split it equally between them. When their mother finally came downstairs she saw that there was only $\frac{1}{4}$ of the pan of brownies left. How much of the pan of brownies did each child eat?
3. In a neighborhood where all the houses are the same, Brad can paint 3 houses in 5 days and Bjorn can paint 2 houses in 3 days. Together, how many houses could they paint in 5 days? How many days would it take them to paint 8 houses?
4. Samuel used a photocopy machine to reduce a drawing to 75% of its original size, and then he lost the original. What enlargement setting should he use to get a copy of the drawing the same size as the original?
5. Brett’s pool is 6 feet longer than it is wide. If a three-foot border around the pool has an area of 192 square feet, what are the dimensions of the pool?
6. The real estate market has fallen 20% since Stephanie bought her house. How much must the market go up for the value of her house to return to the price she paid for it?
7. At Belk’s Super-One-Day-Blue-Dot Sale, Anna decides to buy some shoes that are marked down 25%, and she has a coupon that gives 20% off already-reduced prices. If she opens a Belk’s charge account in order to get an additional 10% off her purchase, what is her total discount before sales tax?
8. When Anna told Ashley about the original price of the shoes and all the discounts, Ashley computed what the non-sale cost would have been with tax, and then she applied all the discounts. Should her computations yield the same total amount that Anna actually paid?
9. If $2\frac{1}{2}$ bags of mulch cover $\frac{3}{4}$ of Sierra’s garden, how many bags would it take to cover the whole garden? How much of the garden does one bag cover?

10. On opening day in the softball season, Rhain got 2 hits in 5 at bats, for an average of $\frac{2}{5}$. On the second day of the season, she got 1 hit in 3 at bats. Kathryn, the team manager, computed Rhain's batting average so far, but Rhain disagreed because that is not how you are supposed to add fractions. Is Kathryn's answer correct?

$$\frac{2}{5} + \frac{1}{3} = \frac{3}{8} = .375$$

11. The perimeter of a rectangle is 17 cm, and its area is 18 cm^2 . Find the dimensions of the rectangle.

$$8\frac{2}{3}$$

12. The product of two numbers is 16, and their sum is $8\frac{2}{3}$. Find the numbers.

13. How many pounds of coffee at \$6.20 per pound must be mixed with 9 pounds worth \$7.10 per pound to make a blend worth \$6.50 per pound?

14. Using a tea that costs 90 cents per ounce and another tea that costs 60 cents per ounce, Jason wants to make a tea blend worth 72 cents per ounce. How many ounces of each tea will she need to make 50 ounces of the blend?

15. Dimpal sells pretzels for \$1.25 each and hotdogs for \$2.00 each. One day she sold 82 items for a total of \$134.75. How many pretzels did she sell?

$$6\frac{2}{3} \times 3\frac{3}{5}$$

16. Compute $6\frac{2}{3} \times 3\frac{3}{5}$ without converting to improper fractions.

17. Brandon calculated $43 \cdot 37 = 40^2 - 3^2 = 1591$. Describe his method. Why does it work? Will the method work for any pairs of whole numbers?

Problems from Sybilla Beckman:

18. To make a shade of orange paint that you like, you must mix $\frac{2}{3}$ of a bottle of red paint with each $\frac{4}{5}$ of a bottle of yellow paint that you use. You need 88 bottles of this orange paint. How many bottles of red paint will you need and how many bottles of yellow paint will you need? (All bottles are the same size.)

19. A flock of geese on a pond were being observed continuously. At 1:00 P.M., $\frac{1}{5}$ of the geese flew away. At 2:00 P.M., $\frac{1}{8}$ of the geese that remained flew away. At 3:00 P.M., 3 times as many geese as had flown away at 1:00 P.M. flew away, leaving 28 geese on the pond. At no other time did any geese arrive or fly away. How many geese were in the original flock?