Math 1165: Supplemental Problems

1. (a) A ball is dropped from 15 feet off the ground and loses 40% of its peak height with every bounce. How far will the ball have traveled if it bounces forever? Ignore concerns about air resistance and friction.

   (b) Another ball is dropped from 15 feet off the ground. Every time it hits the floor it bounces up 120% of its height. How far will the ball have traveled after its 23rd bounce? If it bounces forever? Ignore concerns about air resistance and friction.

2. (An ancient fable): The Persian king has promised you a handsome reward for saving the life of his daughter. You may have one gold piece on the first square of the king's checkerboard, two on the second, four on the third, eight on the fourth, and so on, doubling each time. OR you may have 1000 gold pieces on the first square, 2000 on the second, 3000 on the third, and so on, increasing by 1000 gold pieces each time. How much money do you receive under each reward system?

3. Attendances at an annual corporate picnic have been going up by 5.3% per year over the last several years.

   (a) If the attendance was 430 this year, what will the predicted attendance be five years from now?

   (b) If the attendance was 430 this year, what was the attendance last year?

   (c) If the attendance was 430 this year, what will be the predicted attendance in 2120?

   (d) If the attendance was 430 this year, what was the attendance in 1993?

4. Find the following sums:

   (a) $54.89 + 61.63 + 68.37 + 75.11 + \cdots + 1106.33$

   (b) $3.45 + 2.415 + 1.6905 + 1.18335 + .828345 + \cdots + .0019269832$

5. Do "Summing it all Up", **??** in the Appendix.

6. Juice comes in cans holding z ounces each. Sally was expecting x guests to attend her Super Bowl party and wanted to make sure that

each person (including herself) had y glasses of juice (with each glass holding k ounces). If Sally had n cans of juice at home to begin with, how many cans of juice did Sally need to buy?

7. We want to open a bank account in which we put the same amount of money in the account at the beginning of each year (and no other deposits). If interest is 8% compounded annually and we want to be able to withdraw $100,000 after 8 years, how much should we deposit each year?

8. In each of the following, a loan is described.

    (a) You wish to pay off a loan through annual payments of $3000 for 20 years. If the annual interest rate is 8%, what is the largest loan you can borrow?

    (b) You wish to pay off a $5000 loan in 5 years at an annual interest rate of 7%. What should your annual payments be? In the first payment, how much of it went toward interest and how much went toward paying the $5000?

    (c) You wish to pay off a $5000 loan in 5 years with annual payments of $1500. What is the highest interest rate you can assume that would make this possible?

    (d) You wish to pay off a $50000 loan with payments of $8000 at an annual interest rate of 7%. How long will it take you to pay off the loan?What is the smallest annual payment that is possible for this loan and this interest rate (e.g., what happens if you try to pay off the loan with annual payments of 50?)

9. After spending 2/5 of his money on a toy car, Sammy had $42 left. How much money did he have at first?

    (a) Solve this problem with the aid of a diagram. Explain your solution.

    (b) Now solve the problem algebraically, letting x stand for the amount of money Sammy had to start with.

    (c) Write an expression in terms of x for the amount of money Sammy had left after buying the toy car. Relate your expression to your diagram in part (a).

    (d) Write an equation involving x which states that Sammy has $42 left after buying his toy. Relate your equation to your diagram in part (a).

(e) Solve your equation in part (d). Relate your work to the work you did with your diagram in part (a).

10. There was some flour in a bag. First, $\frac{3}{8}$ of the flour in the bag was used. Then $\frac{1}{5}$ of the remaining flour was used. At that point, there were 10 cups of flour in the bag. Determine how much flour was in the bag at the start by using a picture. Then solve using equations that fit with your picture solution.

11. Susan and Sally had an equal amount of money each. After Sally spent $15 and Susan spent $24, the ratio of Sally's money to Susan's money was 4 : 3. How much money did each girl have at first? Solve this problem using pictures and equations. Explain how they are related.

12. A box is to be constructed by cutting squares out of the corners of a 20 by 30 inch piece of cardboard and folding up the corners. If one cuts out squares of side length x inches,

    (a) Write a formula for the the area of the resultant piece of cardboard and

    (b) Write a formula for the volume of the resultant box.

13. Stupid Number Tricks: Think of a number. Multiply by 5. Add 8. Multiply by 4. Add 9. Multiply by 5. Subtract 105. Divide by 100. Subtract 1. How does the result compare with your original number? Show why this works in general.

14. Finish what we did not do on the activity Graphic Details.

15. While driving to OSU the other day, I traveled out of my subdivision in 3 minutes, waited for traffic for 1 minute, turned onto a 55 mph road and traveled on it for 4 minutes, stopped at a light for 2 minutes, then turned into and traveled on an exit ramp for 30 seconds, sped up and traveled on I-70 (65 mph) for 5 minutes, and slowed down for a traffic jam and wound up in stop-and-go traffic for 3 minutes. Sketch a graph of my distance traveled versus time as well as a graph of my speed versus time (on separate sets of axes).

16. Solve the equation $8x + 6 = 2x + 7$ algebraically through 3 steps: bringing constants together on one side, bringing variables together on one side, and undoing multiplication. When doing each step, show graphically the lines representing each side of the expression. What is different in each step? What is the same in each step?

17. Suppose $f(x) = x^3 - 2x$. What is

   (a) $f(2)$?
   (b) $f(-2)$?
   (c) $f(-3)$?
   (d) $f(f(3) + f(-2))$?
   (e) $f(y/2)$?
   (f) $f(y)/2$?
   (g) $f(\sqrt{3})$?
   (h) $f(-1/2)$?
   (i) $f(2x)$?
   (j) $f(f(f(1)))$?

18. In each case, find $g(5)$ and when $g(x) = 5$, $g(-x) = 5$, $g(x) = x$, $g(x) = 1 + x$.

   (a) $g(x) = 3x + 2$
   (b) $g(x) = 10/(x - 3)$

19. Roger likes to run each morning. He brings along a clock and a pedometer to check the time and how far he has gone.

   (a) On Monday, he started out at 6AM and sees that at 6:20 he's gone 4 miles, what will the pedometer read at 6:50?

   (b) If, on Tuesday, he again starts out at 6AM and his pedometer reads 3.64 miles at 6:23AM, what will the pedometer read at 6:54AM?

   (c) On Wednesday, Roger forgot what time he started. However, he reads on his pedometer that at 6:04 he had gone 4.83 miles and at 6:19 he had gone 7.51 miles. How far will he had gone 51 minutes into his run? At what time would he have gone 2 miles?

20. Martha is the chief hippopotamus caretaker at the Wild Animal Park in San Diego. She has just arrived at the cargo dock in order to pick up four members of the endangered species hippopotamus mathematicus that were recently rescued from African poachers. Before the animals are released by the harbormaster, Martha must weigh them. BUT the only scale big enough to weigh a hippo is the truck scale that doesn't

weigh anything lighter than 300 kilograms (kg); this is more than each hippo weighs. Martha is puzzled for a few minutes, then gets the idea of weighing the hippos in pairs, thinking that if she gets the mass of each possible pair, she can later figure out the masses of the individual hippos. She weighs the hippos pair-by-pair and gets 312 kg, 356 kg, 378 kg, 444 kg, and 466 kg. When she tries to weigh the heaviest pair of hippos, the scale breaks.

(a) What was the mass of the last pair of hippos who broke the scale?

(b) What are the masses of the individual hippos?

NOTE: Be sure to write down explicitly any assumptions you make, and be able to explain your reasoning at each step.

21. Suppose that the gas mileage of a vehicle is linearly related to its weight. A vehicle weighing 2100 pounds receives 34 miles per gallon while a vehicle weighing 5100 pounds receives 14 miles per gallon.

(a) Draw a graph of the relationship between the weight of a car (x) and its gas mileage (per gallon) (y). Be sure to label and scale your axes!

(b) Write an equation for the relationship.

(c) Use the equation to find what gas mileage a car weighing 3000 pounds should expect.

(d) Use the equation to find how much a car should weigh to get 25 miles per gallon.

(e) Describe what the slope represents in terms of what the variables stand for.

(f) Find and describe what each intercept represents in terms of what the variables stand for.

(g) Crazy Joe wants to buy a 9000 pound truck to rule the highway with. What will his gas mileage be? Comment.

22. Explain why the point-slope form of the equation of the line indeed describes what all points (x, y) on the line must satisfy. Note: A line with slope m that contains the point (x_1, y_1) will have equation $y - y_1 = m(x - x_1)$. This is called the "point-slope form of the line."

23. How many times must you fold an ordinary piece of paper in half before it is so thick it reaches to the moon?

24. How much money would the $24 used to buy Manhattan in the year 1626 be worth today had it been invested at an annual rate of 6%?

25. During their hour play time, the two oldest Brady kids (Greg and Marcia) went to the park to play with their walkie-talkies. They used them for an hour. The next day, the next-oldest kid came along. Because there were only two walkie-talkies, they needed to share so that each possible pair got equal time. The next day, the next-oldest came along. This continued until on the ninth day, all ten kids were there wanting to use the walkie-talkies. How much time did Greg and Marcia spend with each other on the walkie-talkies over the course of the nine days?

26. A gardener has to tend to a series of separate circular flower gardens for the king. The first garden has a diameter of one foot, the second has a diameter of two feet, and each successive garden increases the diameter by one foot. If there are 20 gardens, what is the total square footage the gardener must tend to?

27. Find the total number of gifts given in the song "The Twelve Days of Christmas."